Sequential Trades of Resalable Information

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This is the basic model of other works entitled

1. "A Model of Pricing Data and Their Constituent Variables Traded in Two-Sided Markets with Resale: A Subject Experiment," by Nanba, Ogawa, Watanabe, Hayashi, Sakaji, *Proc. 2022 IEEE Int'l Conference on Big Data*, IEEE Xplore, 3288-3294, 2022.
2. "Resale-Proof Trades of Data under Budget Constraints: A Subject Experiment," by Ogawa and Watanabe, *Proc. 2023 Int'l Conference on Big Data*, IEEE Xplore, 5665-5673, December 2023.

Imagine the following data service,
as a simple familiar example.

Creating a new market for used digital communication devices

- various factors (variables) that determine the price of a used device
  - in-store or online retail prices and specifications of new products
  - transaction information in online auctions of second-hand products, including damage, network restrictions, and so on
  - others
- a set of “processed and summarized” variables (data) of a used device specified for each user
- Each instance of data consists of several variables.

1. Introduction: Research Question

- Data and variables are easily replicable. $\Rightarrow$ not scarce $\Rightarrow$ difficult to put a price on variables and data

How can we price the data and its constituent variables?

1. Introduction: Major Features

a new model from the perspective of cooperative games

1. Muto and Nakayama (1992): resale of information (without production)
2. Watanabe and Muto (2008): disadvantage arising due to not owning information (under patent protection)

The prices of variables are exogenously set at the initial round, and they are updated for the next round based on the outcomes of trades made in the current round.

- Conjecture: backward induction $\Rightarrow$ prices never move in any rounds that proceed after the initial round.
- no budget constraints vs. under budget constraints

1. Introduction: Main Results

- The prices of data and the constituent variables fluctuated over rounds. Resale occasionally happens.
- The prices of data determined by the initial owners under budget constraints remained relatively stable without a drastic increase compared with the prices of those data determined under no budget constraints, although the budget constraints were so loose that it would not be actually a constraint at all for rational traders. the budget constraints.
- The efficiency rates in transactions made under budget constraints were not lower than those rates in transactions made under no budget constraints.

2. Model

- 2 instances of data, each consisting of 3 items of variables, traded in 12 rounds ($r = 0, 1, 2, \ldots, 12$) by 4 traders.
- At each round, the prices of the variables are given to traders and updated for the next round immediately at the end of the round, based on the outcomes of trades.
  - $p_i(r)$: price of variable i in round r
  - replicability $\Rightarrow$ Traders purchase one unit of the variable.
  - The production cost of data is the sum of the expenditure for the constituent variables.
  - $t_j(r)$: price of data j in round r
  - replicability $\Rightarrow$ Users of data j purchase one unit of the data.
  - (under budget constraints) Every agent was faced with a temporary budget constraint that amounted to 250 in each round. Even if there was a surplus budget, it could not t be carried over to subsequent rounds.

- the gross profits for an individual user and those for a non-user of the data satisfy the following relationship:

$$W_j(1) \geq W_j(2) \geq \cdots \geq W_j(n) > L_j(0) \\ \geq L_j(1) \geq L_j(2) \geq \cdots \geq L_j(n-1). \quad (1)$$

  When there are s owners of data j, a user obtains $W_j(s)$ while a non-user obtains $L_j(s)$. (disadvantages for non-users)
- The data can be replicated and resold freely.
  - each owner of the data proposes the price and the number of instances of the data, and
  - traders who do not own the data may purchase the data at the proposed price.
- $t_i(r)$ is the price at which the initial owner sells. The resale process stops for all data ⇒ transactions proceed to the next round.
- Any data expire for the use within each round in order to avoid dynamic competition among instances of data at the markets.

- # of users of data j $(j = 1, 2)$ $\Rightarrow$ demand for variable i
- $q_i(r)$: threshold for variable i in round r. This threshold value is used for updating its price $p_i(r)$ that is set at the market in the next round.
- The initial values of $q_i(0)$ and $p_i(0)$ are exogenously given.
- Demand for variable i in round t exceeds $q_i(t)$ $\Rightarrow$ the price in round $t+1$ is updated to $p_i(t+1) = p_i(t) + a_i$. The threshold in round $t+1$ is also updated to $q_i(t+1) = q_i(t) + b_i$.
- Demand falls below $q_i(t)$. $\Rightarrow$ $p_i(t+1) = \max(0, p_i(t) - a_i)$ and $q_i(t+1) = \max(0, q_i(t) - b_i)$.

  $\cdots$ a_i and b_i are positive constants exogenously given.

3. Experimental Design: Gross Profits

Data 1 and Data 2 are traded.

The gross profits are specified as follows.

Data 1: $W_1(1) = 200$, $W_1(2) = 150$, $W_1(3) = 70$, $W_1(4) = 50$, $L_1(0) = 40$, $L_1(1) = 30$, $L_1(2) = 20$, $L_1(3) = 10$.

Data 2: $W_2(1) = 200$, $W_2(2) = 120$, $W_2(3) = 70$, $W_2(4) = 50$, $L_2(0) = 40$, $L_2(1) = 30$, $L_2(2) = 20$, $L_2(3) = 10$.

- complete information environment: the values of gross profits are shown to the subjects.
- incomplete information environment: subjects are informed of the orders of those gross profits. In practice, traders have to estimate those values.

3. Experimental Design: Demands for Variables

of users of data j ($j = 1, 2$) $\Rightarrow$ demand for variable i

First, find the largest integer e such that $2 \leq e \leq 3$ and

$$W_j(e) > W_j(4) + (4 - e)(W_j(4) - L_j(e)).$$

- When $e = 3$, $W_j(3) = 70$ and $W_j(4) + (4 - 3)(W_j(4) - L_j(3)) = 90$. Namely, when there are 3 data holders, each one obtains 70 but one can obtain 90 by reselling the data, but the amount is at most 90.
- When $e = 2$, $W_1(2) = 150$ and $W_2(2) = 120$, but $W_j(4) + (4 - 2)(W_j(4) - L_j(2)) = 110$ for $j = 1, 2$. Namely, when there are 2 data holders, everyone has no incentive to resell the data.

The solution of the above problem is thus $e = 2$.

Second, consider a situation in which there are 2 data holders.

- If one of the data holders resells to a trader, then he can receive $W_j(3) - L_j(2) = 50$ for Data $j = 1, 2$ from the trader.
- In the case of $e = 3$, however, another resale is expected to occur, as shown previously. Thus, the data holders can eventually obtain at most $W_j(3) - L_j(2) + W_j(4) + (W_j(4) - L_j(3)) = 140$ for Data $j = 1, 2$.
- In the case of Data 1, any resale should not occur when $e = 2$, because $W_1(2) = 150$.
- Even in the case of Data 2, any resale should not occur when $e = 2$; $W_2(2) = 120$. When a resale to a non-holder occurred, another resale should occur as shown above, and thus the first reseller would obtained $W_2(3) - L_2(2) + W_2(4) = 100$. Even if the first resale was made to 2 hon-holders, he would obtain $2(W_2(4) - L_2(2)) + W_2(4) = 110$.

Resale does not occur when there are 2 data holders.

Finally, compare the case of $e = 2$ and $e = 1$.

- If the initial holder of Data 1 does not sell the data, then he obtains $W_1(1) = 200$, but he can obtain
  $W_1(2) + (2-1)(W_1(2) - L_1(1)) = 270$
  through monetary transfer by selling the data to a trader.
- Similarly, consider the case of Data 2. $W_2(1) = 200$, but he can obtain $W_2(2) + (2-1)(W_2(2) - L_2(1)) = 210$
  through monetary transfer by selling the data to a trader.

According to the backward induction, 2 data holders should not resell the data to any traders.

3. Experimental Design: Social Welfare

Compute the maximal amounts of producer surplus. (The payments are cancelled between payers and recipients, when summing up their profits.)

- Data 1: the total sum of profits (producer surplus) is maximized at $e = 2$; $2W_1(2) + 2L_1(2) = 2 * 150 + 2 * 20 = 340$.
- Data 2: it is maximized at $e = 1$;
  $W_2(1) + 3L_2(3) = 200 + 3 * 30 = 290$.
  $(2W_2(2) + 2L_2(2) = 2 * 120 + 2 * 20 = 280)$

  Therefore, in terms of “social welfare” of traders, the initial holder of Data 2 should not sell Data 2 to any users.

3. Experimental Design: Theoretical Predictions

- payoff maximization: The initial holders of Data 1 and Data 2 should sell their data to a trader, respectively. Resale should not occur.
- social welfare: It is maximized when there are 2 data holders in the case of Data 1, while it is maximized when the initial holder of Data 2 does not sell the data to any traders.

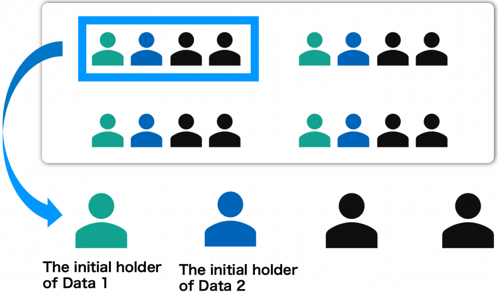
3. Experimental Design: Session Details

We conducted 4 experimental sessions, each consisting of 12 rounds.

	comp xy	comp xz	incomp xy	incomp xz
Data 1	x , y , z	x , y , z	x , y , z	x , y , z
Data 2	x , y	x , z	x , y	x , z
information	complete	complete	incomplete	incomplete

- The initial prices of variables x, y and z : $p_1(0) = 10$, $p_2(0) = 20$, $p_3(0) = 30$.
- $q_1(0) = q_2(0) = q_3(0) = 2$ for Data 1, and $q_1(0) = q_2(0) = 2$ or $q_1(0) = q_3(0) = 2$ for Data 2 (by the previous backward induction)
- The constants for updating those prices and thresholds are $a_1 = a_2 = 10$ and $b_1 = b_2 = 10$, respectively.

In every session, 16 subjects divided into 4 groups of 4 subjects at the beginning of the session.



The initial holder of each data is determined randomly and does not change in all rounds.

- Data 1 $\Rightarrow$ Data 2: The transactions of Data 2 proceed in a manner similar to those for Data 1.
- After the transactions for Data 1 and Data 2, the price of the variables are updated.
- Then, the transactions for the next round start in a similar manner.

Resale can be made at most twice.

The payoff for each agent is

$$\begin{aligned} & [\text{gross profit obtained from Data 1} \\ & + \ \text{(re)selling price(s) of Data 1} - \text{buying price of Data 1}] \\ + \ & [\text{gross profit obtained from Data 2} \\ & + \ \text{total (re)selling price(s) of Data 2} - \text{buying price of Data 2}], \end{aligned}$$

where the buying price of Data j ($j = 1, 2$) is the sum of prices of Data j's constituent variables for the initial owner of Data j.

4. Result: Prices of Variables

Result 1

The average prices of variables $i = x, y, z$ were not far from the initial price $p_i(0)$. In the complete information environment, the variances were not significantly larger in all cases except variable y in session comp xy, as compared with those prices under no budget constraints.

session	variable x	variable y	variable z
comp xy	8.889 (9.428)	**13.472** (9.952)	**23.472** (11.279)
comp xz	9.500 (1.126)	**22.167** (7.831)	**25.000** (12.003)
incomp xy	**13.646** (8.719)	21.458 (12.813)	29.271 (11.262)
incomp xz	12.708 (7.852)	**22.292** (6.270)	29.583 (13.831)

TABLE I: Average prices of the variables under budget constraints.

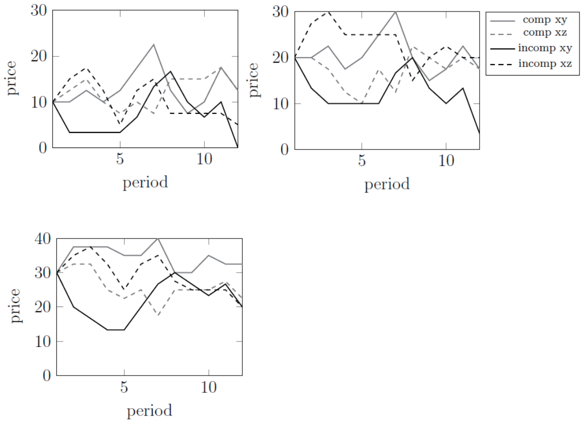


Fig. 3: Prices of variables traded under no budget constraints.

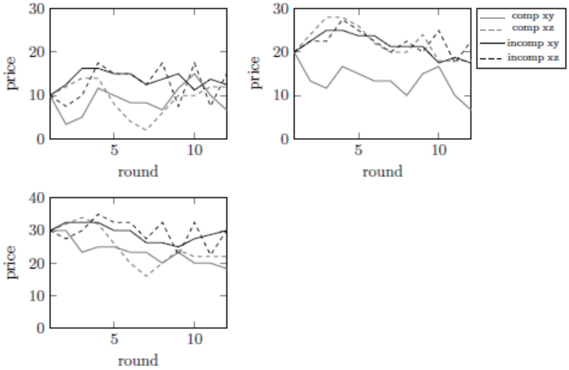


Fig. 4: Prices of variables traded under budget constraints.

4. Result: Prices of Data

Result 2

The prices of data determined by the initial owners under budget constraints remained relatively stable without a drastic increase compared with those determined under no budget constraints.

session	Data 1	Data 2	p-value
comp xy	95.758 (38.793)	93.111 (51.899)	0.395
comp xz	92.889 (39.910)	83.618 (21.500)	0.694
incomp xy	101.961 (37.197)	92.978 (40.622)	0.089
incomp xz	78.450 (26.330)	79.568 (34.506)	0.574

TABLE II: Average prices of data determined by the initial owners under budget constraints.

session	Data 1	Data 2
comp xy	0.473	0.772
comp xz	$p <$**0.001**	$p <$**0.001**
incomp xy	**0.031**	**0.613**
incomp xz	**0.044**	$p <$**0.001**

TABLE III: P-values for the Brunner-Munzel tests: budget constraints vs. no budget constraints.

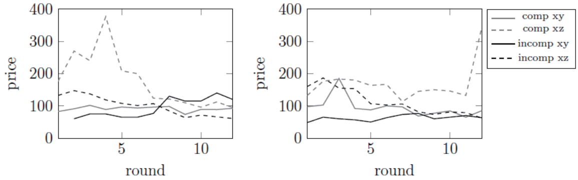


Fig. 5: Prices of data determined by the initial owners under no budget constraints.

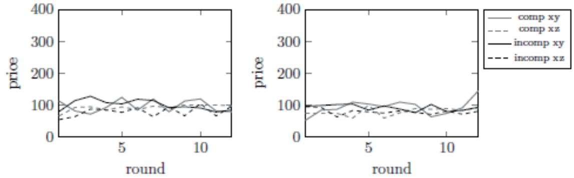


Fig. 6: Prices of data determined by the initial owners under budget constraints.

4. Result: Social Welfare 1

Result 3

For Data 2, the efficiency rates observed in the incomplete information environment differed significantly from those observed in the complete information environment, although there was no such difference for Data 1.

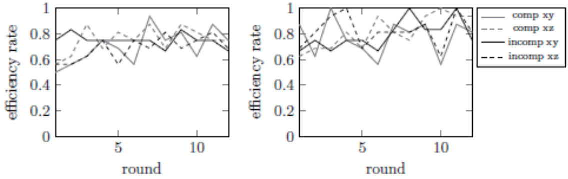


Fig. 7: Average efficiency rates: under no budget constraints.

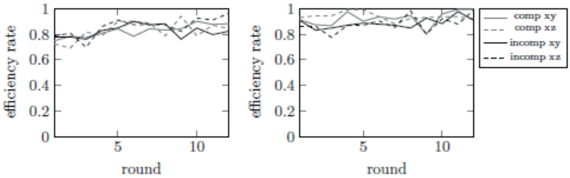


Fig. 8: Average efficiency rates: under budget constraints.

sessions pooled	complete info	Data 1	rounds
comp xy and incomp xy	0.167	**p<0.001**	
	0.191	**p<0.001**	0.062
comp xz and incomp xz	0.345	**0.006**	
	0.345	**0.006**	0.437

TABLE IV: The p-values for a two-way ANOVA.

	Data 1	Data 2	p-value	# of samples
comp xy	1.187	1.393	**p<0.001**	72
incomp xy	1.213	1.299	**0.026**	96
p-value	0.912	**0.022**		
comp xz	1.246	1.422	**0.001**	60
incomp xz	1.296	1.303	0.862	48
p-value	0.280	**0.012**		

TABLE V: Average transformed efficiency rates and p-values for the Brunner-Munzel test.

transformed efficiency rate = arcsin(sqrt(efficiency rate))

In transactions made under no budget constraints, the transformed efficiency rates were 1.058 and 1.276 for Data 1 and Data 2, respectively.

4. Result: Social Welfare 2

Result 4

The efficiency rates in transactions made under budget constraints were not lower than those rates in transactions made under no budget constraints.

session	Data 1	Data 2
comp xy	0.847	0.701
comp xz	0.763	0.864
incomp xy	0.668	0.145
incomp xz	**0.042**	**0.014**

TABLE VI: P-values for the Brunner-Munzel test.

Conducted to compare the efficiency rates of each type of data between results under no budget constraints and under budget constraints.

4. Result: What Factors Made Efficiency Rates Lower?

Result 5

Social welfare declines as the frequency of resale increases.

	resales	Data 1	Data 2
comp xy	0	1.243 (n=59)	1.480 (n=58)
	1	0.930 (n=13)	1.039 (n=12)
	2	—— (n=0)	0.980 (n=2)
p-value		p <**0.001**	p <**0.001**
incomp xy	0	1.304 (n=74)	1.447 (n=62)
	1	0.926 (n=14)	1.043 (n=27)
	2	0.874 (n=8)	0.980 (n=7)
p-value		p <**0.001**	p <**0.001**
comp xz	0	1.371 (n=44)	1.478 (n=53)
	1	0.909 (n=12)	1.025 (n=3)
	2	0.874 (n=4)	0.980 (n=4)
p-value		p <**0.001**	p <**0.001**
incompxz	0	1.371 (n=40)	1.382 (n=38)
	1	0.926 (n=7)	1.009 (n=8)
	2	0.874 (n=1)	0.980 (n=2)
p-value		p <**0.001**	p <**0.001**

TABLE VII: Averages transformed efficiency rates and p-values of the Kruskal-Wallis test.

5. Final Remarks: Main Results

- The prices of data and the constituent variables fluctuated over rounds. Resale occasionally happens.
- The prices of data determined by the initial owners under budget constraints remained relatively stable without a drastic increase compared with the prices of those data determined under no budget constraints, although the budget constraints were so loose that it would not be actually a constraint at all for rational traders. the budget constraints.
- The efficiency rates in transactions made under budget constraints were not lower than those rates in transactions made under no budget constraints.

5. Final Remarks: For Future Research

- Conducting additional treatment.
  - more realistic budget constraints
- Extension to a more general model.
  - We need to introduce asymmetric gross profits in to our model.
- How to prevent from selling false contents intentionally.

Appendix 1: The Basic Model w/o Production and Budget

- $N = \{1, 2, \ldots, n\}$: the finite set of players, where player 1 is an initial owner of information, and players 2, ..., and n are its demanders.
- $W(m)$ ($L(m)$): the gross profits to each informed (uninformed) player, when the information is shared by m players;

$$W(1) \geq W(2) \geq \ldots, \geq W(n) > L(0) \geq L(1) \geq \ldots, \geq L(n-1) \geq 0.$$

ref. Cournot Nash equilibrium profits are summarized in the following order:
$W(1) > \cdots > W(n) > L(0) > \cdots > L(K) = \cdots = L(n-1) = 0.$

- the initial state $[\{1\}; x^0]$, where $x_0 = (x_i^0)_{i \in N}$ is a vector of payoffs given by

$$x_i^0 = \begin{cases} W(1) & \text{for } i = 1; \\ L(1) & \text{for any } i \neq 1 \end{cases}$$

When the information is traded within a group of players $\{1\} \cup S$, where $S \subseteq N \setminus \{1\}$, denote by p_i the amount of money that player i gains from or pays to members of $\{1\} \cup S$ and $s = |S|$. Let $y = (y_{i \in \{1\} \cup S})$ be a vector of payoffs in $\{1\} \cup S$, where

$$y_i = W(1+s) + p_i.$$

We say that a vector y of payoffs is an $\{1\} \cup S$-imputation at the initial state $[\{1\}; x^0]$ if the following conditions are satisfied;

$\sum_{i \in \{1\} \cup S} p_i = 0$ i.e., $\sum_{i \in \{1\} \cup S} y_i = (1+s)W(1+s)$ (balancedness in $\{1\} \cup S$)

and

$y_i \geq x_i^0$ for any $i \in \{1\} \cup S$ (individual rationality of $i \in \{1\} \cup S$).

Sps: members in $\{1\} \cup S$ reach a particular $\{1\} \cup S$-imputation y^* in negotiations and they share the information as a result.

a new state $[\{1\} \cup S; x^1]$: $\{1\} \cup S$ is the set of informed players and $x^1 = (x_i^1)_{i \in N}$ is the vector of payoffs given by

$$x_i^1 = \begin{cases} y_i^* = W(1+s) + p_i^* & \text{for any } i \in \{1\} \cup S; \\ L(1+s) & \text{for any } i \notin \{1\} \cup S. \end{cases}$$

If $\{1\} \cup S = N$, then trading is over. Otherwise, resale by a member of $\{1\} \cup S$ may occur.

- At the state $[\{1\} \cup S; x^1]$, a group of players Q consisting of both informed and uninformed players, i.e., $Q \cap (\{1\} \cup S) \neq \emptyset$ and $Q \cap (N \setminus (\{1\} \cup S)) \neq \emptyset$, starts negotiations on resale.
  - $T = Q \cap (\{1\} \cup S)$: the set of informed players in Q
  - $R = Q \cap (N \setminus (\{1\} \cup S))$: the set of uninformed players in Q.

Let $y = (y_i)_{i \in Q}$ be a payoff vector in Q which is given by

$$y_i = \begin{cases} W(1+s+r) + p_i^* + q_i & \text{for any } i \in T; \\ W(1+s+r) + q_i & \text{for any } i \in R, \end{cases}$$

where $s = |S|$, $r = |R|$, and q_i is the amount of money that player i gains from or pays to members of Q.

We say a payoff vector y is a *Q-imputation at the state* $[\{1\} \cup S; x^1]$, if it satisfies the balancedness in Q and individual rationality of $i \in Q$;

$$\sum_{i \in Q} q_i = 0 \text{ and } y_i \geq x_i^1 \text{ for any } i \in Q.$$

Sps: members in Q agree upon a particular Q-imputation y^*, and they share the information.

another state $[\{1\} \cup S \cup R; x^2]$, where $x^2 = (x_i^2)_{i \in N}$ is the payoff vector given by

$$x_i^2 = \begin{cases} y_i^* = W(1+s+r) + p_i^* + q_i^* & \text{for any } i \in T; \\ y_i^* = W(1+s+r) + q_i^* & \text{for any } i \in R; \\ W(1+s+r) + p_i^* & \text{for any } i \in (\{1\} \cup S) \setminus T; \\ L(1+s+r) & \text{for any } i \in (N \setminus (\{1\} \cup S)) \setminus R. \end{cases}$$

1. If members in Q can not agree upon a particular Q-imputation y^*, then resale process stops and the state $[\{1\} \cup S; x^1]$ is the outcome of this sequential trades.
2. If $Q = N$ and members in Q agree upon some Q-imputation y^*, then trading is over.
3. Otherwise, resale process may continue.

Appendix 2: Solution Concept

- A state $[M; x]$, where $M = \{1\} \cup S$, is stable if for any objection of an arbitrary player $i \in Q$ against another player $j \in Q \setminus \{i\}$ in x, there exists a counter objection of j against i.
- Namely, a variant of (Aumann-Maschler) bargaining set.

Our solution concept is as follows;

A state $[M, x]$ is stationary, if no group $Q \in \triangle(M)$ has a stable Q-imputation at $[M, x]$, where
$$\triangle(M) = \{Q \subseteq N : Q \cap M \neq \emptyset \text{ and } Q \cap (N \setminus M) \neq \emptyset\}.$$

We define our solution concept considering the resale process backwardly.

Formal Definitions:

- Suppose that all players have obtained player 1's information. Let $x = (x_i)_{i \in N}$ be a payoff vector, where $x_i = W(n) + p_i$ and p_i denotes the net amount of money that player i has gained or paid up to that time. A payoff vector x is balanced in N if

$$\sum_{i \in N} p_i = 0.$$

If all players obtained the information and an associated payoff vector is balanced, this state will last.

Thus, we say that the state $[N; x]$ is stationary for each balanced payoff vector x associated with N.

- Denote a state by $[M; x]$, where M is a set of informed players, and x is a vector of payoffs given by

$$x_i = \begin{cases} W(m) + p_i & \text{for any } i \in M; \\ L(m) & \text{for any } i \in N \setminus M, \end{cases} \tag{2}$$

which meets the balancedness in M:

$$\sum_{i \in M} p_i = 0, \text{ or } \sum_{i \in M} x_i = mW(m).$$

Here, p_i is the net amount of money that i gained or paid before the state is reached.

Let $\triangle(M) = \{Q \subseteq N : Q \cap M \neq \emptyset \text{ and } Q \cap (N \setminus M) \neq \emptyset\}$. For each $Q \in \triangle(M)$, let

$$Q^M = Q \cap M \text{ and } Q^{-M} = Q \cap (N \setminus M),$$

and let y^Q be a vector of payoffs in Q given by

$$y_i = \begin{cases} W(m + q^{-M}) + p_i + p_i^Q & \text{for any } i \in Q^M; \\ W(m + q^{-M}) + p_i^Q & \text{for any } i \in Q^{-M}, \end{cases}$$

where $q^{-M} = |Q^{-M}|$ and p_i^Q is the amount of money that i gains from or pays to the members in Q.

We say that a vector of payoffs y^Q is a Q-imputation at $[M, x]$ if $\sum_{i \in Q} p_i^Q = 0$ and $y_i^Q \geq x_i$ for any $i \in Q$.

For each $Q \in \triangle(M)$, if all members of Q agree upon a Q-imputation y^Q and resale is carried out, then the information is shared by members of $M \cup Q^{-M}$ and a new vector of payoffs $z = (z_i)_{i \in N}$ is given by

$$z_i = \begin{cases} W(m + q^{-m}) + p_i + p_i^Q & \text{for any } i \in Q^M; \\ W(m + q^{-m}) + p_i^Q & \text{for any } i \in Q^{-M}; \\ W(m + q^{-m}) + p_i & \text{for any } i \in M \setminus Q^M; \\ L(m + q^{-m}) & \text{for any } i \in (N \setminus M) \setminus Q^M. \end{cases}$$

When $\sum_{i \in M} p_i = 0$ and $\sum_{i \in Q^{-M}} p_i^Q = 0$, z is a vector of payoffs that is balanced in $M \cup Q^{-M}$. Thus, we have a new state $[M \cup Q^{-M}, z]$.

- For each $Q \in \triangle(M)$, we say that a Q-imputation y^Q at $[M, x]$ is valid if a new state $[M \cup Q^{-M}, z]$ induced by y^Q is stationary.
- What is stationarity? That will be defined recursively in what follows.
  - The state $[N; x]$ is stationary for each balanced payoff vector x associated with N.
  - We should consider the resale process backwardly from $[N; x]$.
- Take a valid Q-imputation y^Q and take two members i and j of Q arbitrarily. We say that i has an objection (K, y^K) against j in y^Q, if there exists a set $K \in \triangle(M)$ with $i \in K$ and $j \notin K$ and a valid K-imputation y_i^Q such that

$$y_i^K > y_i^Q,$$
$$y_j^K \geq y_j^Q \text{ for any } i \in K \cap (Q \setminus \{i\}).$$

For this objection, we say that j has a counter objection (L, y^L) against i, if there exists a set $L \in \triangle(M)$ with $i \notin L$ and $j \in L$ and a valid L-imputation y^L such that

$$\begin{aligned} y_i^L &\geq y_i^K \quad \text{for any } i \in K \cap L, \\ y_j^L &\geq y_j^Q \quad \text{for any } j \in (L \setminus K) \cap Q. \end{aligned}$$

We say that a valid Q-imputation y^Q is stable, if for each $i, j \in Q$ $(i \neq j)$ and each objection of i against j at y^Q, there exists a counter objection of j against i.

A state $[M, x]$ is *stationary*, if no group $Q \in \triangle(M)$ has a stable Q-imputation at $[M, x]$.

Appendix 3: Results

We here consider the highest resale prices of the information.

The definition of $m(r)$:

Let $m(0) = n$.
Then, let $m(1)$ be the largest integer m that satisfies $2 \leq m \leq m(0) - 1$ and

$$W(m) > W(n) + (m(0) - m)(W(n) - L(m)). \tag{3}$$

Further let $m(2)$ be the largest integer that satisfies $2 \leq m \leq m(1) - 1$ and

$$W(m) > W(m(1)) + (m(1) - m)(W(m(1)) - L(m)),$$

i.e.,

$$W(m) - L(m) > (m(1) - m + 1)(W(m(1)) - L(m))$$

Define $m(3)$, ..., in a similar manner.

In general, $m(r)$ is defined as the largest integer that satisfies $2 \leq m(r) \leq m(r-1) - 1$ and

$$W(m) - L(m) > (m(r-1) - m + 1)(W(m(r-1)) - L(m)). \qquad (4)$$

Let $m(r^*)$ be the minimum integer of $m(r)$.

- For any state $[M, x]$, where $\{1\} \subset M \subseteq N$ and x is a vector of payoffs that is balanced in M, if $|M| = m(r)$ for some $r = 1, \ldots, r^*$, then $[M, x]$ is stationary; otherwise, not.

Example:

The gross profits are specified as follows.

$W(1) = 200,\ W(2) = 120,\ W(3) = 70,\ W(4) = 50,\ L(0) = 40,$
$L(1) = 30,\ L(2) = 20,\ L(3) = 10.$

First, find the largest integer e such that $2 \leq e \leq 3$ and

$$W(e) > W(4) + (4 - e)(W(4) - L(e)).$$

- When $e = 3$, $W(3) = 70$ and $W(4) + (4 - 3)(W(4) - L(3)) = 90$. Namely, when there are 3 informed players, each obtains 70 but one can obtain at most 90 by reselling the information.
- When $e = 2$, $W(2) = 120$, but $W(4) + (4 - 2)(W(4) - L(2)) = 110$ for $j = 1, 2$. Namely, when there are 2 informed players, everyone has no incentive to resell the data.

The solution of the above problem is thus $e = 2$.

Second, consider a situation in which there are 2 data holders.

- $W(2) = 120$.
- If an informed player resells to an uninformed player, expecting to receive $W(3) - L(2) = 50$ from him, another resale should occur, as shown above. Then, the first reseller would obtained $W(3) - L(2) + W(4) = 50 + 50 = 100$.
- Even if the first resale was made to 2 uninformed platyers, he could obtain only $2(W(4) - L(2)) + W(4) = 2 * 30 + 50 = 110$.

Resale does not occur when there are 2 informed players.

Finally, compare the case of $e = 2$ and $e = 1$.

- $W(1) = 200$, but the initial holder of information can obtain $W(2) + (2-1)(W(2) - L(1)) = 210$ through monetary transfer by selling the data to an uninformed player.
- Choose $e = 2$.

We found that the information will be sold to an uninformed player. After that, Both of 2 informed players will not resell the information to any uninformed players.

Proposition

(1) Suppose that $W(1) < m(r^*)W(m(r^*))$. *Then, when* $m(r^*) = n$, *then the information will be eventually shared by all players, i.e., Only the state* $[N, x^N]$ *is stationary state, where* x^N *is an arbitrary N-imputation in* $[N, x]$. *When* $m(r^*) < n$, *the information will be shared by* $m(r^*)$ *players, i.e.,* $[M, y]$ *is the stationary state, where* $|M| = m(r^*)$ *and* y *which gives the highest possible payoff for player 1 is as follows;*

$$y_i = \begin{cases} m(r^*)(W(m(r^*) - L(m(r^*))) & \text{for } i = 1; \\ L(m(r^*)) & \text{for any } i \in N \setminus \{1\}. \end{cases}$$

(2) Suppose that $W(1) \geq m(r^*)W(m(r^*))$. *Then, the information will be kept by player 1. Namely, the initial state* $[\{1\}, x^0]$ *is stationary.*