Farsighted Stability in Patent Licensing

Toshiyuki Hirai[1], Naoki Watanabe[2], Shigeo Muto[3]

[1]Toyama U., [2]Keio U., [3]Tokyo U. Sci

December 20, 2018 at Hitotsubashi University

An Abstract Game Approach:

We define absolute maximality for abstract games, which was originally defined by Ray and Vohra (2018) for TU games, and investigate whether we can refine farsightedly stable sets in patent licensing.

- Absolute maximality plays a role of something like subgame perfection in extensive form games.
- Some discussions are needed for it. (Today's goal)

Introduction

We consider a negotiation on patent licensing among farsighted

- one (external) patent holder (PH),
- (symmetric) firms.

The PH and firms (players) negotiate on

- which firms are licensed,
- how much fees for the license are.

- Whether an agreement on patent licensing can be reached.
- What properties the agreement has, *e.g.*
  - how much profit the patent holder can obtain,
  - how many firms are licensed.

Patent licensing problem

Non-cooperative game approach

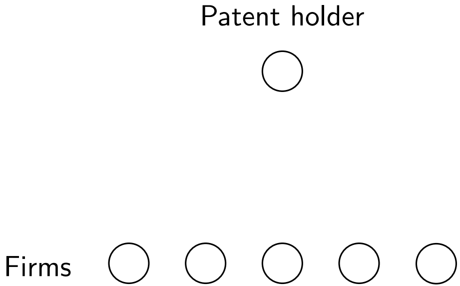
- Kamien and Taumann (1984,1986)

Licensing agreements are contract terms.
→ Cooperative game approach (ref. Tauman and Watanabe, 2007)

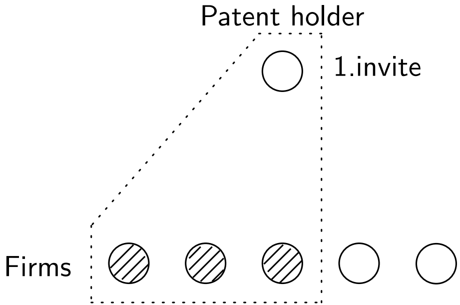
- Watanabe and Muto (2008).
  - Watanabe and Muto (2008): Core; Bargaining set.
  - Kishimoto *et al.* (2011): Shapley value.
  - Kishimoto and Watanabe (2017): Kernel; Nucleolus.
  - Hirai and Watanabe (2018): vNM stable set.

We incorporate players' farsightedness into the model of Watanabe and Muto (2008) with some modification.

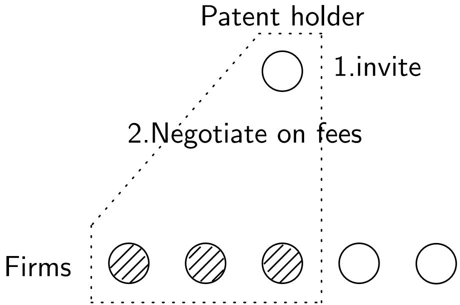
Introduction (WM2008 model)



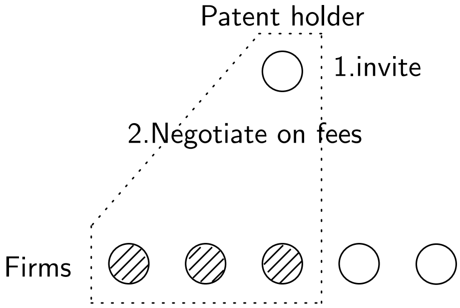
Introduction (WM2008 model)



Introduction (WM2008 model)

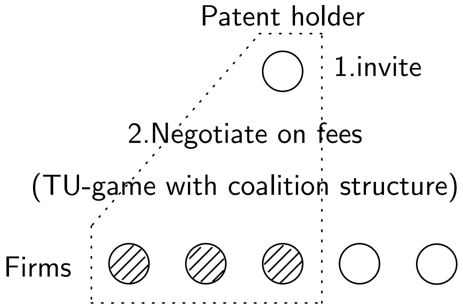


Introduction (WM2008 model)



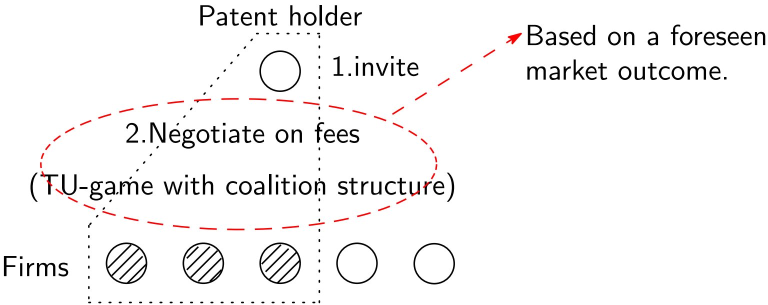
3.All firms compete in a market,
knowing which firms are licensed.
No cartel is allowed.

Introduction (WM2008 model)

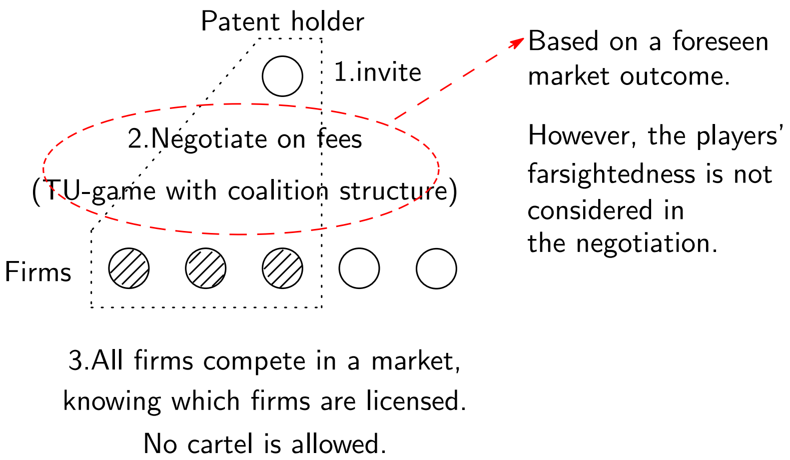


3.All firms compete in a market,
knowing which firms are licensed.
No cartel is allowed.

Introduction (WM2008 model)



3.All firms compete in a market, knowing which firms are licensed.
No cartel is allowed.


Patent holder
1.invite
2.Negotiate on fees
(TU-game with coalition structure)
Firms
3.All firms compete in a market,
knowing which firms are licensed.
No cartel is allowed.
Based on a foreseen
market outcome.
However, the players'
farsightedness is not
considered in
the negotiation.

Introduction

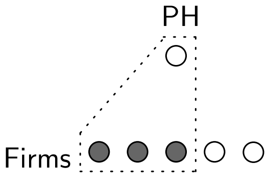
We incorporate farsightedness of the players in the licensing negotiation.

- An analogous motivation of Diamantoudi (2005).

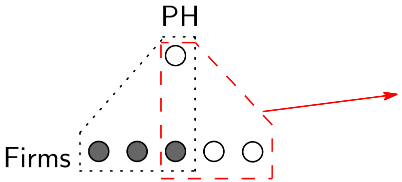
We modify the model of Watanabe and Muto (2008) so that

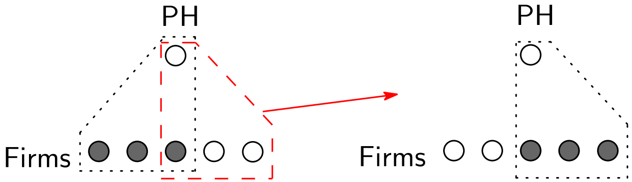
1. PH and firms negotiate
   - not only on contracts of licensing fees,
   - but also which firms are licensed.
2. Firms compete in a market after patent licensing.
   - Firms commonly know which firms are licensed.
   - No cartel is allowed.

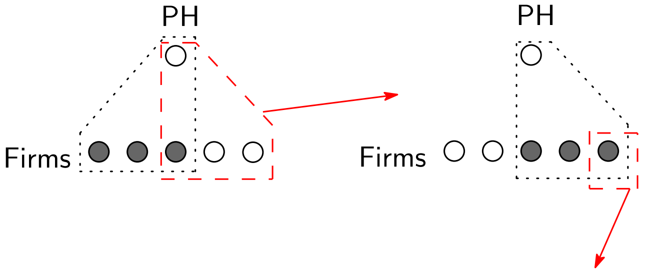
Introduction



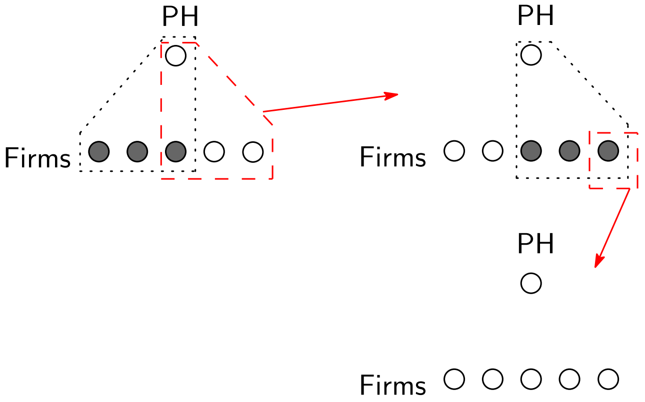
Introduction

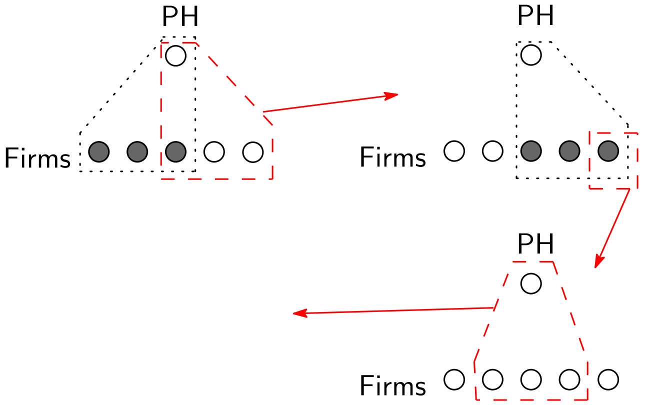



PH
Firms
PH
Firms

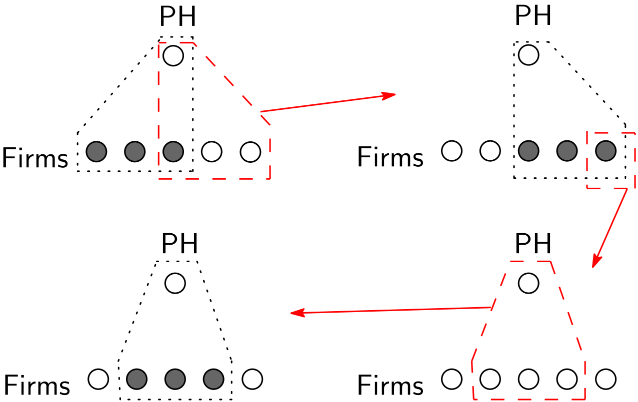

PH
Firms
PH
Firms

Introduction

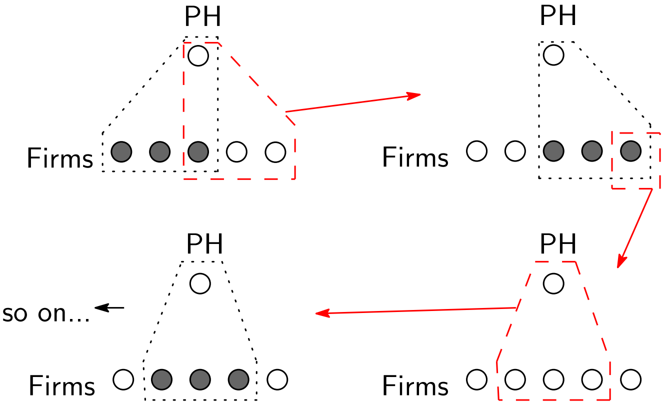



PH
Firms
PH
Firms
PH
Firms

Introduction



Introduction



Introduction

We investigate farsighted stable sets (Harsanyi, 1974; Chwe, 1994) in an abstract game describing such a negotiation.

- In particular, we focus on symmetric farsighted stable sets.
  - A farsighted stable set where at each outcome in the set, licensee firms obtain symmetric payoffs.

Main results

- We characterize symmetric farsighted stable sets under certain conditions.
  - The number of licensee firms that maximizes the profit of PH, provided that each licensee firm receives an exogenously given positive net profit.
  - The existence result follows from this characterization.
  - Maximality problem is also considered.

Model: players

- 0: the patent holder (PH).
  - The PH has no production technology.
- $N = \{1, ..., n\}$: the set of firms.
  - Potential users of the patented technology.
  - Firms are symmetric if they are not licensed.
  - Licensee firms become symmetric.
- $\{0\} \cup N$: the set of players.
- A nonempty subset of $\{0\} \cup N$: a coalition.

Model: profits at market competition

Firms compete in a market after the patent licensing negotiation.

When s $(0 \leq s \leq n)$ firms are licensed,

- each licensee firm obtains $W(s)$;
- each non-licensee firm obtains $L(s)$.
  - $L(0)(> 0)$: the profit of firms when no firm is licensed.
  - $W(0) = L(n) = 0$ by convention.

Assumption

(a) $W(s) > L(0)$ *for all* $s = 1, ..., n$;
(b) $L(0) > L(s)$ *for all* $s = 1, ..., n - 1$.

Model: outcomes (1)

An outcome of the negotiation is represented by

- a set of licensee firms;
- a contract of profit sharing.

The set of feasible contracts when $S \subseteq N$ is licensed: $(s = |S|)$

$$X^S = \left\{ x = (x_0, x_1, \dots, x_n) \in \mathbb{R}^{n+1} \;\middle|\; \begin{array}{l} x_0 + \sum_{i \in S} x_i = |S| W(|S|), \\ x_j = L(|S|) \text{ for all } j \in N \setminus S \end{array} \right\}$$

- $X^{\emptyset} = \{(0, L(0), \dots, L(0))\} \equiv \{x^{\emptyset}\}$.
  - $x^{\emptyset}$: null contract.
  - $(\emptyset, x^{\emptyset})$: null outcome.

Model: outcomes (2)

The set of all outcomes:

$$X = \bigcup_{S \subseteq N} (\{S\} \times X^S).$$

The set of symmetric outcomes:

$$\bar{X} = \{(S, x) \in X | \ x_i = x_j \text{ for all } i, j \in S\}.$$

- The licensee firms pay an uniform fee at a symmetric outcome.

Model: effectiveness relation

An effectiveness relation describes what a coalition can induce an outcome from an outcome.

- $(S, x) \rightarrow_T (S', x')$;
- Coalition T can induce (S', x') from (S, x).

Assumption

Let $(S, x), (S', x') \in X$ *with* $S' \neq \emptyset$.
(i) $(S, x) \rightarrow_T (\emptyset, x^{\emptyset})$ *if and only if* $\emptyset \neq T \subseteq \{0\} \cup S$;
(ii) $(S, x) \rightarrow_T (S', x')$ *if and only if* $T = \{0\} \cup S'$.

- (i) Any participant of a contract can cancel it unilaterally.
- (ii) A new contract is available with consents from all of PH and licensee firms.

We inherit the spirit of the coalitional game model by Watanabe and Muto (2008).

Indirect dominance relation

Definition

Let $(S, x), (S', x') \in X$.
(S', x') indirectly dominates (S, x), denoted by $(S', x') \succ (S, x)$, iff there exist

- $(S^0, x^0), \ldots, (S^m, x^m)$ and
- $T^1, \ldots, T^m$

such that $(S^0, x^0) = (S, x)$, $(S^m, x^m) = (S', x')$, and for all $h = 1, \ldots, m$,

- $(S^{h-1}, x^{h-1}) \rightarrow_{T^h} (S^h, x^h)$;
- $x'_i > x_i^{h-1}$ for all $i \in T^h$.

We sometimes write the paths like

$$(S^0, x^0) \rightarrow_{T^1} (S^1, x^1) \rightarrow_{T^2} \cdots \rightarrow_{T^{m-1}} (S^{m-1}, x^{m-1}) \rightarrow_{T^m} (S^m, x^m)$$

Indirect dominance relation

Definition

Let $(S, x), (S', x') \in X$.
(S', x') indirectly dominates (S, x), denoted by $(S', x') \succ (S, x)$, iff there exist

- $(S^0, x^0), \ldots, (S^m, x^m)$ and
- $T^1, \ldots, T^m$

such that $(S^0, x^0) = (S, x)$, $(S^m, x^m) = (S', x')$, and for all $h = 1, \ldots, m$,

- $(S^{h-1}, x^{h-1}) \rightarrow_{T^h} (S^h, x^h)$;
- $x'_i > x_i^{h-1}$ for all $i \in T^h$.

We sometimes write the paths like

$$(S^0, x^0) \rightarrow_{T^1} (S^1, x^1) \rightarrow_{T^2} \cdots \rightarrow_{T^{m-1}} (S^{m-1}, x^{m-1}) \rightarrow_{T^m} (S^m, x^m)$$

$<_{T^1}$ (S^m, x^m) $\quad <_{T^2}$ (S^m, x^m) $\quad <_{T^m}$ (S^m, x^m)

Farsighted stable set

Definition

- $K \subseteq X$ is a farsighted stable set iff internal and external stabilities are satisfied.

  Internal stability: for any $(S, x), (S', x') \in K$,
  $(S', x') \succ (S, x)$ does not hold.

  External stability: for any $(S, x) \in X \setminus K$,
  $(S', x') \succ (S, x)$ for some $(S', x') \in K$.

- $K \subseteq X$ is a symmetric farsighted stable set iff K is a farsighted stable set and $K \subseteq \bar{X}$.

Main results: preparation

Define

$$E = \{\varepsilon \in \mathbb{R}_{++} \, | s(W(s) - L(0) - \varepsilon) > 0 \text{ for some } s = 1, ..., n\}.$$

The set of net profits of licensee firms from patent licensing that are strictly acceptable for PH.

- $E \neq \emptyset$ by $W(s) > L(0)$ for all $s = 1, ..., n$.

For each $\varepsilon \in E$, define

$$B(\varepsilon) = \arg \max_{s=1,...,n} s(W(s) - L(0) - \varepsilon).$$

The optimal number(s) of licensee firms for PH under $\varepsilon \in E$.

Main results: sufficiency

For any $\varepsilon \in E$, define

$$\bar{X}(\varepsilon) = \left\{ (S, x) \in \bar{X} \;\middle|\; \begin{array}{l} |S| \in B(\varepsilon) \\ x_0 = |S|(W(|S|) - L(0) - \varepsilon) \\ x_i = L(0) + \varepsilon \text{ for all } i \in S \end{array} \right\}.$$

Theorem 1

For each $\varepsilon \in E$, $\bar{X}(\varepsilon)$ is a symmetric farsighted stable set.

- In a farsighted stable set $\bar{X}(\varepsilon)$, net profit $\varepsilon > 0$ of licensee firms is exogenously given.
- An established order of society or accepted stable standard of behavior. (von Neumann and Morgenstern, 1944).

Corollary 1

There exists a symmetric farsighted stable set. (By $E \neq \emptyset$.)

Main results: necessity

Theorem 2

Assume that $L(s)$ is nonincreasing in s. If K is a symmetric farsighted stable set, then $K = \bar{X}(\varepsilon)$ for some $\varepsilon \in E$.

Corollary 2

Assume that $L(s)$ is nonincreasing in s. Then, K is a symmetric farsighted stable set iff $K = \bar{X}(\varepsilon)$ for some $\varepsilon \in E$.

- Nonincreasingness of $L(s)$ holds, when *e.g.*
  - the patented technology is a cost reduction technology, and
  - the market competition is a Cournot competition with linear demand and cost functions.

Comparison with other solutions for the TU game

In the present paper, the profits of PH x_0 supported by symmetric farsighted stable sets are

$$0 < x_0 < \max_{s=1,\ldots,n} s(W(s) - L(0)).$$

- The profits of PH supported by symmetric farsighted stable sets cover the ones supported by the bargaining set.
  - The supremum is consistent with the bargaining set by Watanabe and Muto (2008).
  - The infimum is consistent with the bargaining set only if $n \in \arg\max_{s=1,\ldots,n} s(W(s) - L(0))$.
- No clear relationship with vNM stable set: farsightedness vs. myopia? Let x_0 be the profit of PH in vNM stable set.
  $sW(s) + (n-s)L(s) - nL(0) \leq x_0$
  $\leq \max_{t=0,\ldots,n-s} t(W(t) - L(s)).$

Sketch of proof: Theorem 1

Let $\varepsilon \in E$. We show the "external stability" of $\bar{X}(\varepsilon)$.

- Fix an arbitrary $(T, y) \in X \setminus \bar{X}(\varepsilon)$.
- Let $s^* \in B(\varepsilon)$.
  - $s^*(W(s^*) - L(0) - \varepsilon)(> 0)$ is the profit of PH in $\bar{X}(\varepsilon)$.
- Recall $x^{\emptyset} = (0, L(0), ..., L(0))$.

Case 1. $T = \emptyset$. $(\Rightarrow (T, y) = (\emptyset, x^{\emptyset}).)$

$$(\emptyset, x^{\emptyset}) \rightarrow_{\{0\}\cup S} (S, x)$$

yields $(S, x) \succ (\emptyset, x^{\emptyset})$ for any $(S, x) \in \bar{X}(\varepsilon)$ by

- $x_0 > 0 = x_0^{\emptyset}$
- $x_i = L(0) + \varepsilon > L(0) = x_i^{\emptyset}$ for all $i \in S$.

Case 2. $T \neq \emptyset$ and $y_0 < s^*(W(s^*) - L(0) - \varepsilon)$.

$$(T, y) \rightarrow_{\{0\}} (\emptyset, x^{\emptyset}) \rightarrow_{\{0\} \cup S} (S, x)$$

yield $(S, x) \succ (T, y)$ for any $(S, x) \in \bar{X}(\varepsilon)$ by

- $x_0 = s^*(W(s^*) - L(0) - \varepsilon) > y_0$,
- $x_0 > 0 = x_0^{\emptyset}$,
- $x_i = L(0) + \varepsilon > L(0) = x_i^{\emptyset}$ for all $i \in S$.

Case 3. $T \neq \emptyset$ and $y_0 \geq s^*(W(s^*) - L(0) - \varepsilon)$.

By $(T, y) \notin \bar{X}(\varepsilon)$, there exists $j \in T$ with $y_j < L(0) + \varepsilon$.
Otherwise,

- $y_0 \geq s^*(W(s^*) - L(0) - \varepsilon)$,
- $y_i \geq L(0) + \varepsilon$ for all $i \in T$, and
- $s^* \in B(\varepsilon)$

imply $(T, y) \in \bar{X}(\varepsilon)$, a contradiction.

Let $(S', x') \in \bar{X}(\varepsilon)$, where $j \in S'$.

$$(T, y) \rightarrow_{\{j\}} (\emptyset, x^{\emptyset}) \rightarrow_{\{0\} \cup S'} (S', x')$$

yield $(S', x') \succ (T, y)$ by

- $x'_j = L(0) + \varepsilon > y_j$,
- $x'_0 > 0 = x^{\emptyset}_0$,
- $x'_i = L(0) + \varepsilon > L(0) = x^{\emptyset}_i$ for all $i \in S'$.

Maximality problem

Recent literature discusses the credibility of indirect dominance relation.

- Ray and Vohra (2014, 2018), Dutta and Ray (2017), Dutta and Vartiainen (2017).
- Among others, we examine the absolute maximality by Ray and Vohra (2018) in our farsighted stable sets.

$$(S^0, x^0) \longrightarrow_{T^1} (S^1, x^1) \longrightarrow_{T^2} (S^2, x^2)$$

Maximality problem

Recent literature discusses the credibility of indirect dominance relation.

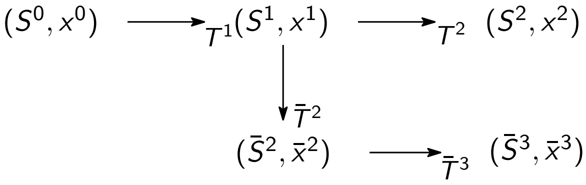
- Ray and Vohra (2014, 2018), Dutta and Ray (2017), Dutta and Vartiainen (2017).
- Among others, we examine the absolute maximality by Ray and Vohra (2018) in our farsighted stable sets.

$$
\begin{array}{ccccc}
(S^0, x^0) & \xrightarrow{}_{T^1} & (S^1, x^1) & \xrightarrow{}_{T^2} & (S^2, x^2) \\
 & & \big\downarrow \bar{T}^2 & & \\
 & & (\bar{S}^2, \bar{x}^2) & &
\end{array}
$$

Maximality problem

Recent literature discusses the credibility of indirect dominance relation.

- Ray and Vohra (2014, 2018), Dutta and Ray (2017), Dutta and Vartiainen (2017).
- Among others, we examine the absolute maximality by Ray and Vohra (2018) in our farsighted stable sets.



$\bar{T}_2$ prefers $(\bar{S}^3, \bar{x}^3)$ to (S^2, x^2).

On Maximality

Not for the farsighted stable set, but for the players' payoffs

- credibility of indirect domination

Ray and Vohra (2018) showed that in TU games,

- a necessary and sufficient condition for a farsighted stable set to be absolutely maximal.

We confirmed the absolute maximality for abstract games.

- Is $\bar{X}(\varepsilon)$ absolutely maximal?

History

A history h is $((S^0, x^0), (T^1, (S^1, x^1)), \dots, (T^m, (S^m, x^m)))$ such that

- if $(S^{\ell-1}, x^{\ell-1}) \neq (S^\ell, x^\ell)$, then $(S^{\ell-1}, x^{\ell-1}) \rightarrow_{T^\ell} (S^\ell, x^\ell)$, where $T^\ell \neq \emptyset$,
- if $(S^{\ell-1}, x^{\ell-1}) = (S^\ell, x^\ell)$, then $T^\ell = \emptyset$.

A history consisting of a single outcome ($m = 0$) is called an initial history.

- An initial history is unnecessarily consisting of $(\emptyset, x^\emptyset)$.

For any history h, let $(S(h), x(h))$ denote the last outcome in h.

- When $h = ((S^0, x^0), (T^1, (S^1, x^1)), \dots, (T^m, (S^m, x^m)))$, then $(S(h), x(h)) = (S^m, x^m)$.

(Negotiation) process

For any history h, define a function σ that gives

$$\sigma(h) = (R(h), (Q(h), z(h))) \text{ such that } (S(h), x(h)) \rightarrow_{R(h)} (Q(h), z(h)).$$

A process prescribes

- an outcome $(Q(h), z(h))$ that follows h,
- a coalition that induces $(Q(h), z(h))$ from $(S(h), x(h))$.

An outcome (S, x) is absorbing under a negotiation process σ, if for any history h with $(S(h), x(h)) = (S, x)$, $(Q(h), z(h)) = (S(h), x(h)) = (S, x)$.

Absorbing process

For any history h, define $\sigma^1(h) = \sigma(h)$ and $\sigma^k(h) = \sigma(h, \sigma^1(h), ..., \sigma^{k-1}(h))$ inductively for any $k > 1$.

σ is an absorbing process if for any history h, σ^k reaches to some absorbing outcome for a sufficiently large k.

- $(Q^k(h), z^k(h))$ is an absorbing outcome where $\sigma^k(h) = (R^k(h), Q^k(h), z^k(h))$.
- Denote $(S^\sigma(h), x^\sigma(h))$ the absorbing outcome that reaches from h under σ.

Coalitionally acceptable absorbing process

An absorbing process is coalitionally acceptable
if for each history h,

$$R(h) \neq \emptyset \text{ implies } x_i^{\sigma}(h) \geq x_i(h) \text{ for all } i \in R(h).$$

- No incentive for every member of any coalitions appearing in an absorbing process to stop the process.

Absolutely maximal absorbing process

An absorbing process σ is absolutely maximal
if for any history h, there exist no $(T^+, (S^+, x^+))$ such that

- $(S(h), x(h)) \rightarrow_{T^+} (S^+, x^+)$,
- $x_i^\sigma(h^+) > x_i^\sigma(h)$ for all $i \in T^+$,
  where $h^+ = (h, (T^+, (S^+, x^+)))$.

- No coalition has incentive to intervene the process in order to induce another absorbing outcome.

Absolutely maximal farsighted stable set

A farsighted stable set K is absolutely maximal if there is an absorbing, coalitionally acceptable, and absolutely maximal process σ such that

- K is the set of all absorbing outcomes of σ,
- for any initial history $h = (S^0, x^0)$ such that $(S^0, x^0) \notin K$, a history $(h, \sigma^1(h), ..., \sigma^k(h))$ constitutes $(S^\sigma(h), x^\sigma(h)) \succ (S^0, x^0)$;
- for any history $h = ((S^0, x^0), (T^1, (S^1, x^1)))$ such that $(S^0, x^0) \in K$ and $(S^1, x^1) \notin K$, a history $(h, \sigma^1(h), ..., \sigma^k(h))$ constitutes $(S^\sigma(h), x^\sigma(h)) \succ (S^1, x^1)$.

(In both histories, an outcome in $\sigma^k(h)$ is the first absorbing outcome appeared in $(h, \sigma^1(h), ..., \sigma^k(h))$.)

Absolute maximality of $\bar{X}(\varepsilon)$

Theorem 3

For any $\varepsilon \in E$, $\bar{X}(\varepsilon)$ is an absolutely maximal farsighted stable set.

The sketch of the proof of Theorem 1 constructs the absorbing, coalitionally acceptable, and absolutely maximal process for Theorem 3 with some additions.