A School Choice Experiment: Cognitive Ability and Information

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1. Introduction: Today's Research Question

- In a school choice (college admissions) experiment (**Chen-Kesten 2019, GEB**),
  - **How does cognitive ability of subjects and information provided to them affect the allocative efficiency and subjects' behavior?** $\cdots$ stability and equilibria: omitted today
- environments: 4 schools (6 schools) with capacity of 1 and 4 stuednts (6 students); one student has one district school.
- Boston mechanism (Bos)
  vs. deferred acceptance mechanism (DA)
- 4 ($= 2 \times 2$) treatments of information $\times$ 2 groups of subjects in cognitive ability scores
  - complete vs. incomplete info on students' preferences, clear vs. unclear info on schools' priority orders
    CK2019's treatment = complete and unclear info
  - high vs. low scores in Raven's APM test

1. Introduction: Main Results I (4 schools - 4 students)

- Which information is important for the allocative efficiency?
  - Complete or Incomplete Info on the other students' preferences over schools does **not** significantly affect the efficiency under both **Bos** and **DA**.
  - Clear or Unclear Info on schools' priority orders over students **does** affect the efficiency under **DA**.
- Subjects' behavior is sensitive to both types of info. **except high score holders under Bos**.
  - About 50-70% of choices were dominant strategies.
  - About 90% of choices were representing true preferences or choosing district schools as the top choice in **every session**.
- **The result in Chen-Kesten (2019) can be supplemented**: Bos $\prec$ DA in efficiency under CK2019's treatment of information in each group of cognitive ability scores
  - **In CK2019, Bos $\sim$ DA in efficiency**.
  - We could replicate Bos $\sim$ DA by mixing the observed behaviors of 2 high score holders and 2 low score holders.

1. Introduction: Main Results II (6 schools - 6 students)

still in progress... (delayed due to the COVID-19), but we have already had the following result under CK2019' treatment of information.

- **The result in Chen-Kesten (2019) can be supplemented**: Bos $\sim$ DA in efficiency by matching 4 high score holders and 2 low score holders and in the environment of 6 high (low) score holders.
  - **In CK2019, Bos $\succ$ DA in efficiency**.
  - We could replicate Bos $\succ$ DA by matching 2 high (3 high) score holders and 4 low (3 low) score holders.
- $\Rightarrow$ Matching subjects with different groups of cognitive ability score generates different results on allocative efficiency.
  - **We should control subjects' cognitive ability in experiments**.
- In the environment of 6 low score holders, truth-telling was observed less than that observed in the other environment.

2. Experimental Design: Students' Preferences

- Students' preferences over schools are represented by payoffs.
- (4 schools - 4 students) Each student obtains 11 points when he or she is matched with his or her district school (secondly preferred school).
  - Assume that there is a utility function that represents their preference orderings.

Table: Students' preferences.

student \ school	A	B	C	D
1	11	7	5	16
2	5	11	7	16
3	7	16	11	5
4	5	16	7	11

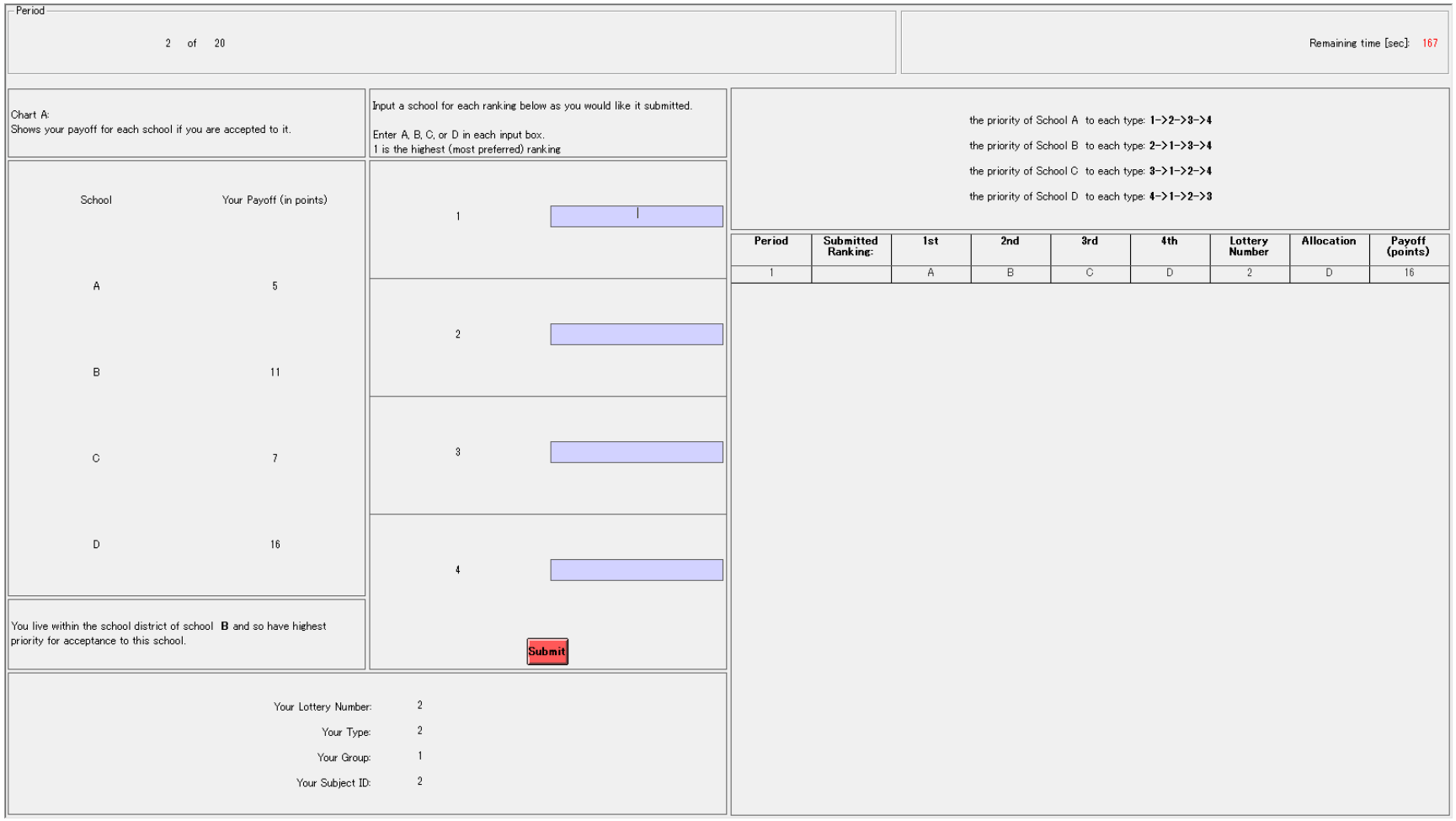
In a complete information treatment, students' preferences are shown the instruction, whereas the others' preferences are not shown there in an incomplete information treatment.

2. Experimental Design: Schools' Priority Orders

- In Chen-Kesten (2019), priority orders over students are determined by one of 4-digit (6-digit) queues.
  1. Each student is given the first priority by his or her district school.
  2. (4 schools - 4 students) Given a 4-digit number, say 2341, school B gives the second priority to student 4 which corresponds to 1 in 2341 (smallest among 2, 4, and 1). Similarly, school B gives the third priority to student 1 which corresponds to 2 in 2341 (smaller between 2 and 4). 2413 is the priority order of students for school B
- In summary, school B gives priorities to students according to

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix},$$

  where the numbers in the first row represent student IDs.
- Subjects have to compute those priority orders by themselves.



In CK2019's treatment of information, priority orders are not displayed on subjects' monitor; unclear information. The above Figure shows a subject's monitor in the treatment with clear information.

go back to Introduction: Another Literature

Pais-Pintér (2008, GEB): Information provision in the case of 5 "teachers" and 3 schools (with priority orders over teachers)

- zero info setting = Incomplete and Uncertain info
- low info setting = Incomplete and Certain info
- partial info setting = Complete and Uncertain info
- full info setting = Complete and Certain info

A surprising result: percentile of choosing the dominant strategy is ordered in zero $>$ low $>$ "partial = full" under both Bos and DA.

- We could not observe this remarkably clear relationship.
- It was, however, partly observed that withholding information induced more subjects to choose dominant strategies (e.g., low score holders under Bos).

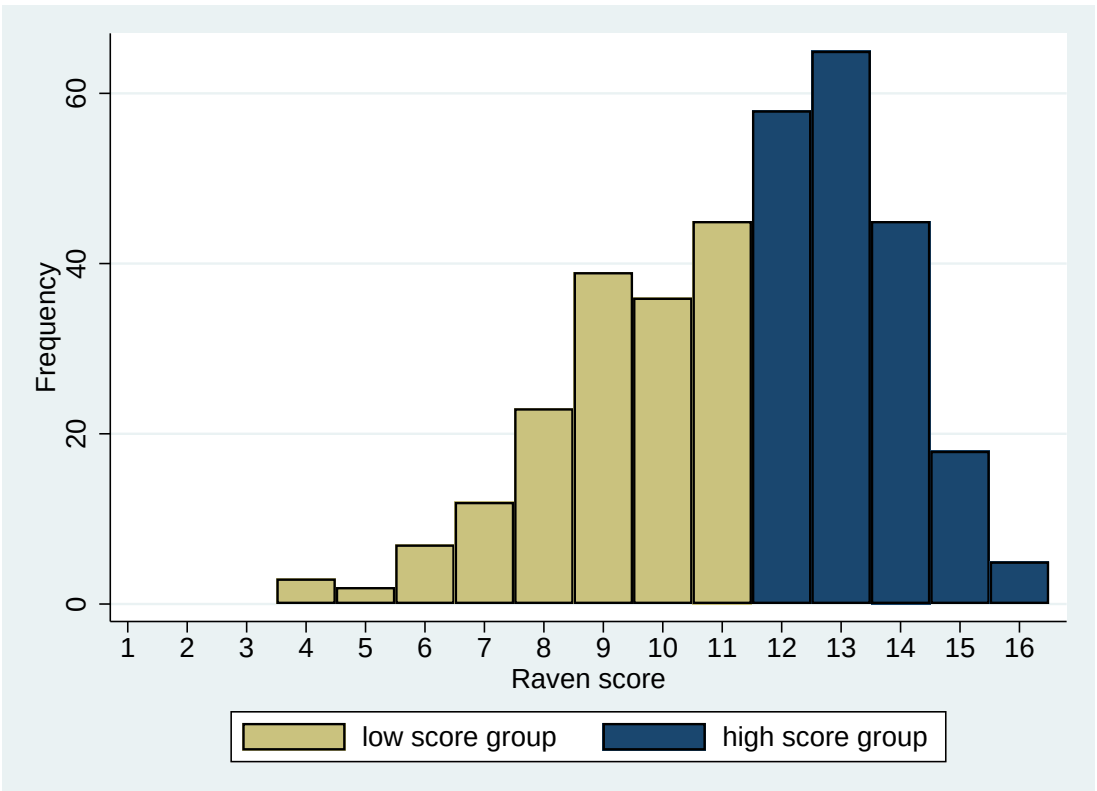
2. Experimental Design: Subject Pool

- Raven test: measuring subjects' ability of pattern recognition.
  - In each question of the test, eight patterns are drawn, and the subject selects a pattern that matches those visual patterns from the options.
  - Colored Progressive Matrices (CPM), Standard Progressive Matrices (SPM), and Advanced Progressive Matrices (APM), in ascending order of difficulty.
  - The APM version of the test is composed of 48 questions in total. We used 16 questions excerpted from the test.
- The Raven scores of the subjects had been taken prior to the dates for our sessions, who were recruited from all over the campus at Kansai University in Japan.
- Our subjects were recruited from that subject pool.
- Subjects were categorized into two groups, high score group (12-16 points) and low score group (4-11 points).

2. Experimental Design: Subjects' Raven Scores

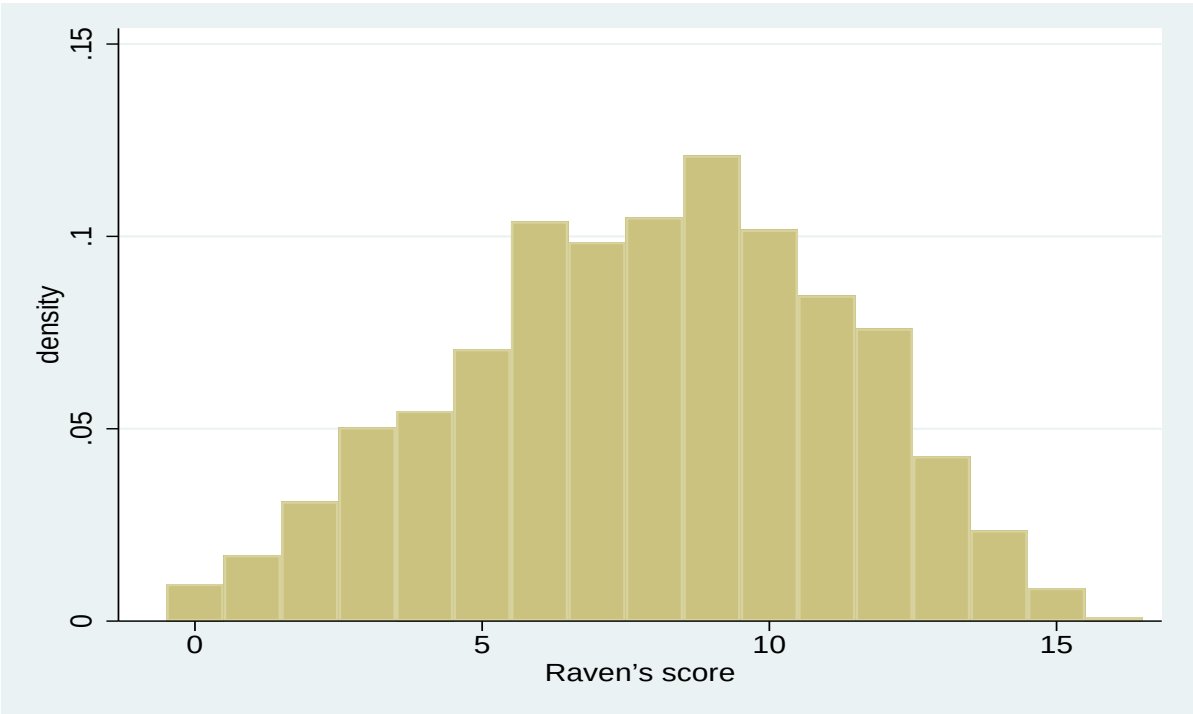
The case of 4 schools and 4 students

	Obs.	Mean score	Standard Deviation	Min	Max
All	384	11.135	2.418	4	16
High	192	13.12	0.982	12	16
Low	192	9.151	1.686	4	11



2. Experimental Design: Raven Score of Non-students

	Num. of Obs.	Mean	Std. Dev.	Min.	Max.
2016.4-2018.3	934	7.935	3.283	0	16



2. Experimental Design: Treatments and Categories

- There are 4 experimental treatments of information: complete and clear info (CC), complete and unclear info (CU), incomplete and clear info (IC), and incomplete and unclear info (IU).
- In each treatment of information, we have 4 environments (2 $\times$ 2 factorial structure).

	Boston	DA
high	12x2 subjects	12x2 subjects
low	12x2 subjects	12x2 subjects

- 384 (=12 subjects x 2 sessions x 4 treatments x 4 environments) participants (4-4 case).
- 180 (=18 subjects x 2 mechanisms x 5 combinations) participants (6-6 case, only CK2019's treatment)
- Each session, which lasted approximately 100 minutes. Every subject had not ever participated in school choice experiments.

go back to Introduction: Subjects' Groups

Baskeck-Mantovani (2018, GEB) first argued the cognitive ability in a school choice (not college admissions) experiment, using the Raven SPM test,

- but their criterion for the classification into 2 groups is not fixed; difference from our subject control.
- our experimental site: Exp. Econ. Lab. at Kansai University has taken the score of a cognitive ability test from almost all subjects since April in 2016; no effect on random recruiting

go back to Introduction: Bos vs. DA

Featherstone-Niederle (2016, GEB): 4 schools with capacity of 1 and 5 students, Incomplete and Clear info, uniform distribution over preferences.

- Bos $\succ$ DA in efficiency.
  - Schools has the same preference for two types of students, Top or Average. (Clear info.)
  - Truth-telling is a unique weakly undominated strategy under Bos. Subjects could learn that truth-telling is better.
- CK2019
  - 4 schools - 4 students; Bos $\sim$ DA in efficiency. District schools are the 2nd preferred ones.
  - 6 schools - 6 students; Bos $\succ$ DA in efficiency. District schools are the 2nd-5th preferred ones. Truth-telling is not necessarily a dominant strategy.
  - Thus, the reasoning behind Bos $\succ$ DA should be different from that derived by Featherstone-Niederle. (See the following slides) $\Rightarrow$ subjects' cognitive ability

3. Part 1

The case of 4 schools and 4 stundents

We could replicate CK2019's result (Bos $\sim$ DA) by mixing the observed behaviors of 2H2L (2 high score holders and 2 low score holders).

3. Part 1: Stable Matching and Nash Eqm

- There exists a unique **stable matching**

$$\mu^1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ A & B & C & D \end{pmatrix},$$

  which is the **Nash equilibrium outcome** under Bos and DA (district schools as the top choices under Bos and truth-telling under DA).
- Under DA, there is the other Nash equilibrium outcome, which is not stable but efficient,

$$\mu^2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ A & D & C & B \end{pmatrix},$$

  which is obtained by $P = (P_1, P_2, P_3, P_4)$, where $P_1 = (A, *, *, *)$, $P_2 = (D, B, *, *)$, $P_3 = (C, *, *, *)$, and $P_4 = (B, D, *, *)$.

$\Rightarrow$ Intuitively, CK2019 would obtain Bos $\prec$ DA, but actually they had Bos $\sim$ DA. subjects' cognitive ability or unclear information?

- Questions from the subjects were answered privately.
- The experiment was programmed in z-Tree (Fischbacher, 2007).
- Each session has 20 rounds in total. Priority orders are changed in every 5 rounds. There is no practice round.
- 14 review questions; the first four are the computation of the matching in Bos or DA (Q1). The answers to those questions are shown in the monitor.
- Subjects are randomly matched every round in such a way that there are 4 types of students in each group.
- Payment: 1000 JPY for show-up +# of correct answers in review questions x 20 JPY + 1 point earned in each round x 5 JPY. Subjects were paid privately in cash.
- average payment = about 2365 JPY.

3. Review Questions

	Bos (Q1)	DA (Q1)	Bos (Q2-Q11)	DA (Q2-Q11)
clear, comp				
high	22	13	0.9167	0.8708
low	21	14	0.7542	0.7792
unclear, comp				
high	23	17	0.9550	0.9250
low	22	14	0.8950	0.8750
clear, incomp				
high	21	12	0.9375	0.9292
low	23	13	0.7375	0.7750
unclear, incomp				
high	22	14	0.8792	0.8208
low	22	8	0.9042	0.8208

3. Part 1: Efficiency

$$\text{normalized efficiency rate} = \frac{\text{max group rank (14)} - \text{actual group rank}}{\text{max group rank (14)} - \text{min group rank (6)}}$$

	Bos mean	(std. dev.)	DA mean	(std. dev.)	p-value (BM)
comp, unclear (CK2019)					
high	0.7531	(0.0573)	0.7875	(0.1106)	$<$**0.0001**
low	0.7167	(0.1231)	0.7573	(0.0693)	$<$**0.0001**
p-value (BM)	**0.0016**		**0.0013**		

- **Observation A**: For both groups of cognitive ability, the efficiency rates under the DA were higher than those under the Bos in CK2019' treatment (Complete and Uncertain info). Subjects' cognitive ability matters.

3. Part 1: Allocation

A matching is observed frequently under Bos

$$\mu^3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ A & C & B & D \end{pmatrix},$$

where the sum of ranks is 8 and the efficiency rate is 0.750.

The efficiency rate of

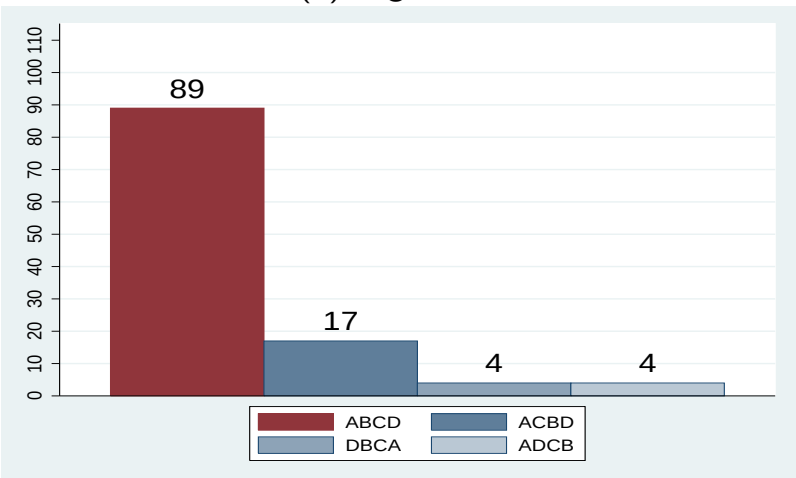
$$\mu^4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ D & B & C & A \end{pmatrix}$$

is 0.625. It is 0.750 for μ^1 and 1.000 for μ^2, respectively.

The sample size = 120 for each category; observed frequencies of equilibria were few.

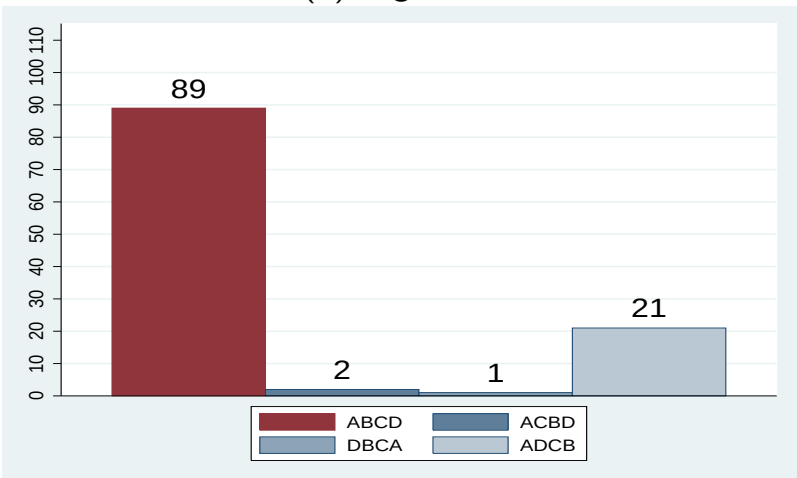
Complete, Unclear (CK2019)

(a) High, Bos



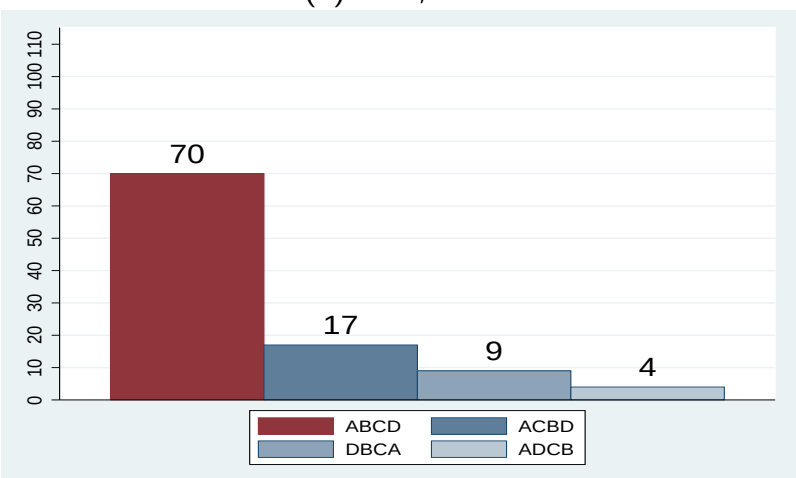
num dist = 12 in ABCD

(b) High, DA



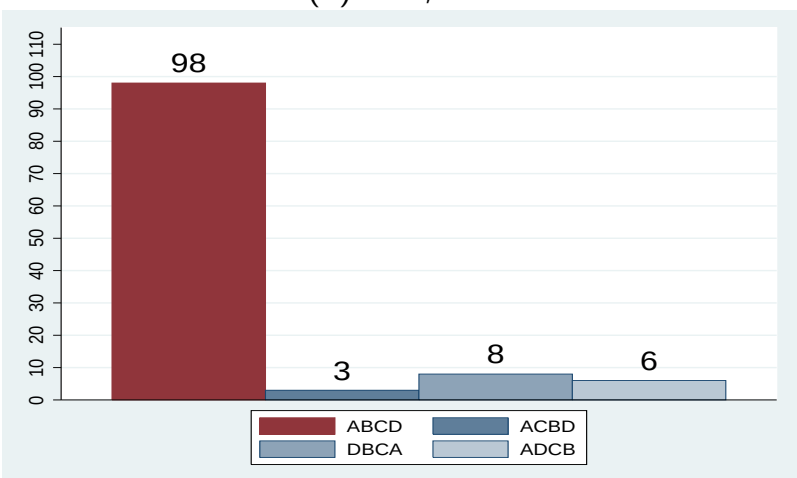
num TT = 38 in ABCD num P in ADCB = 10

(c) Low, Bos



num dist = 5 in ABCD

(d) Low, DA

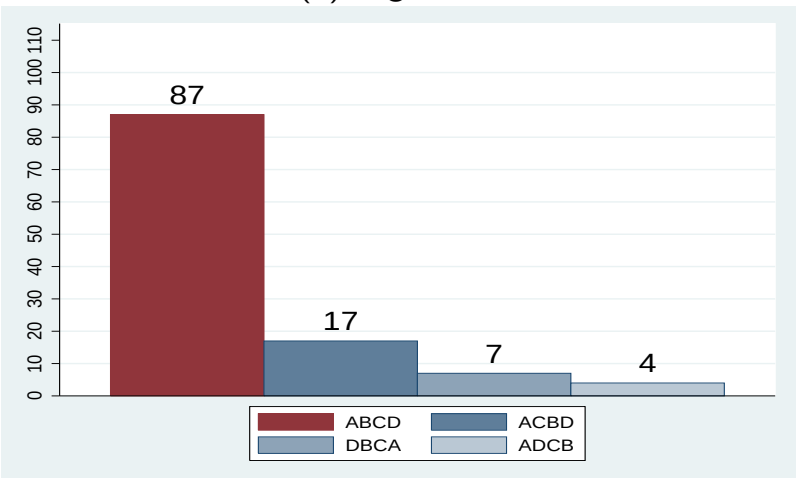


num TT = 18 in ABCD num P in ADCB = 3

The sample size = 120 for each category; observed frequencies of equilibria were few.

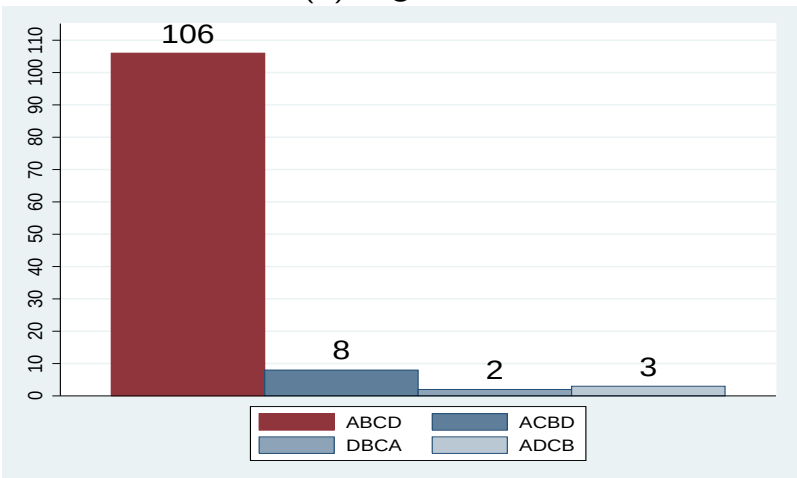
Complete, Clear

(a) High, Bos



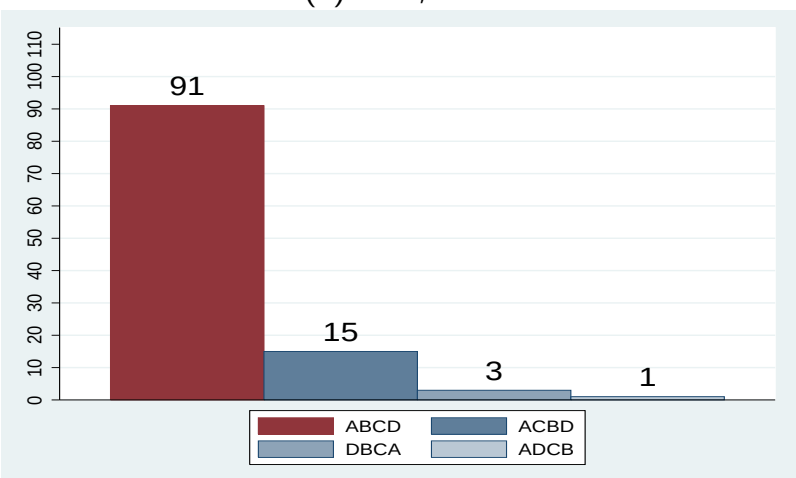
num dist = 8 in ABCD

(b) High, DA



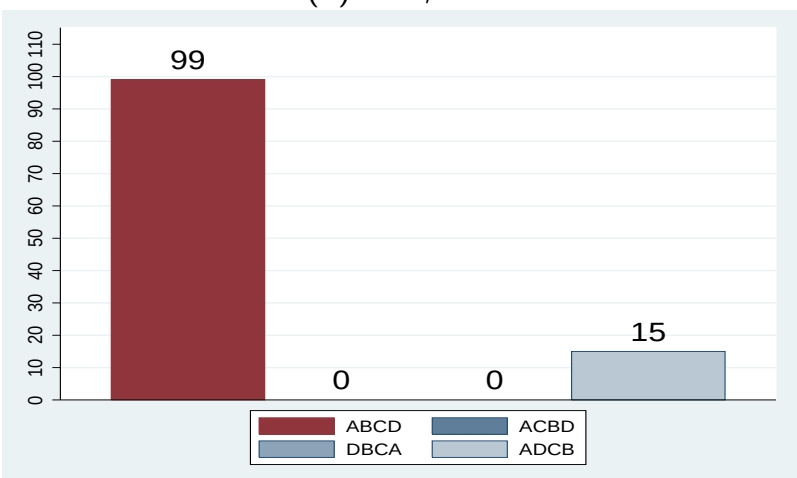
num TT = 31 in ABCD num P in ADCB = 0

(c) Low, Bos



num dist = 16 in ABCD

(d) Low, DA



num TT = 6 in ABCD num P in ADCB = 6

3. Part 1: Sensitivity to Information Provision

Table: P-values for the Shapiro-Wilk test for normality of samples

	Shapiro-Wilk				Levene
	CC	CU	IC	IU	
asin(*sqrt*(*BosHdis*))	**0.0033**	0.1972	0.2230	0.4404	0.6625
asin(*sqrt*(*BosLdis*))	0.4229	0.9143	0.2619	**0.0013**	0.8132
asin(*sqrt*(*DAHtru*))	0.1096	0.1583	0.1753	0.6596	0.6045
asin(*sqrt*(*DALtru*))	0.2968	0.1693	0.3035	**0.0479**	0.9532

- In the Shapiro-Wilk tests, the null hypothesis is that the samples are normally distributed.
- In the Levene tests, the null hypothesis is that the variances of samples are equal among four treatments.

Table: Results of a two-way ANOVA.

	BosHdis	BosLdis	DAHtru	DALtru
	p-value	p-value	p-value	p-value
factor U	0.1217	0.8401	0.0501	**0.0029**
factor I	0.1398	$<$**0.0001**	**0.0121**	$<$**0.0001**
U $\cdot$ I	0.7944	$<$**0.0001**	**0.0039**	$<$**0.0001**

Observation B:

- District schools as the top choice made by high score holders under Bos was not sensitive to information provision.
- low score holders under DA is most sensitive to information provision.

3. Part 2

The case of 6 schools and 6 students

- As for the allocative efficiency and subjects' behavior, we could **replicate** CK2019's result (Bos $\succ$ DA, distribution of TT) in 2H4L (2 high score holders and 4 low score holders).
- Bos $\sim$ DA in 6H, 4H2L, and 6L.

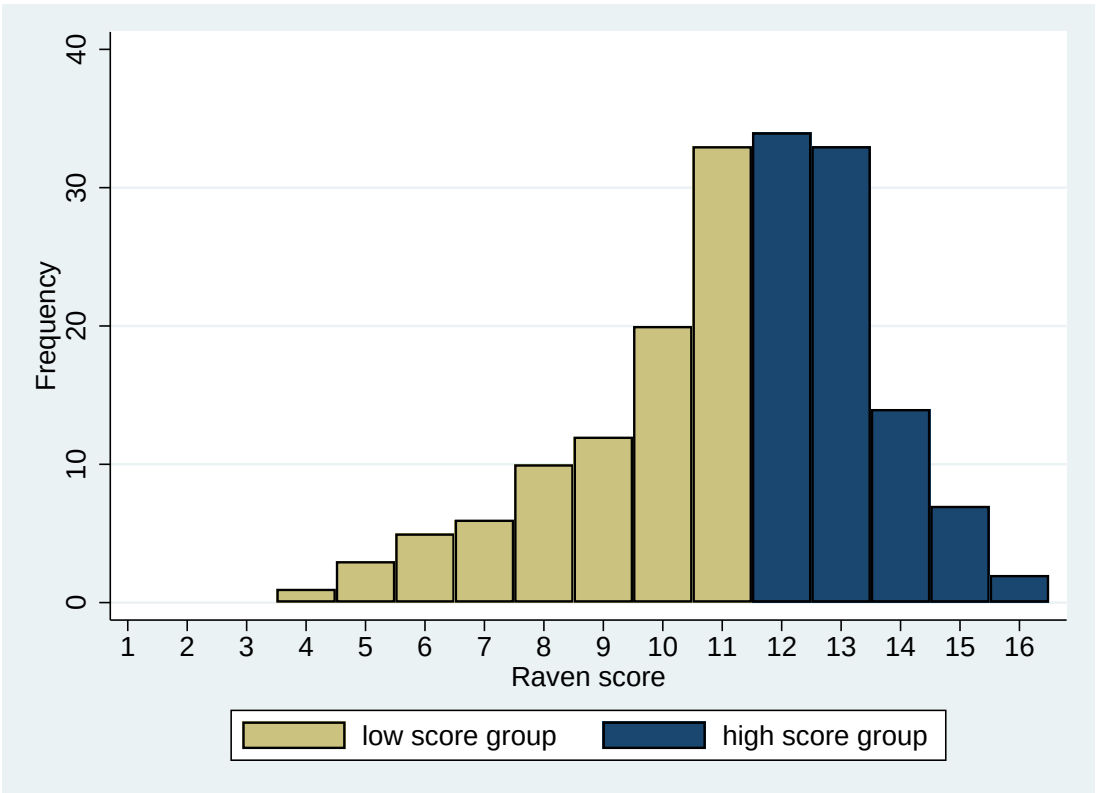
Table: Students' preferences.

student \ school	A	B	C	D	E	F
1	9	16	11	13	7	5
2	16	11	5	13	9	7
3	9	16	7	11	5	13
4	16	7	9	13	5	11
5	16	13	11	7	9	5
6	16	13	11	5	7	9

District schools are 2nd to 5th preferred by students in this environment.

3. Raven score in the case of 6 schools and 6 students

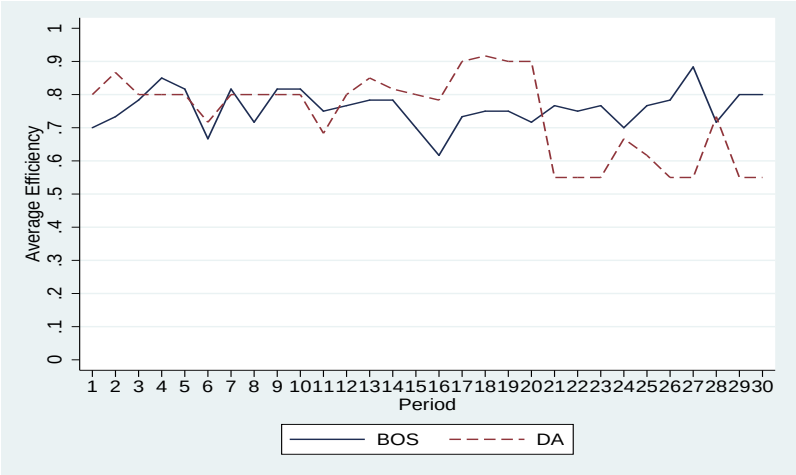
	Obs.	Mean score	Standard Deviation	Min	Max
All	180	11.178	2.345	4	16
High	90	13.000	1.028	12	16
Low	90	9.356	1.813	4	11



3. Results 2. 6 schools and 6 students

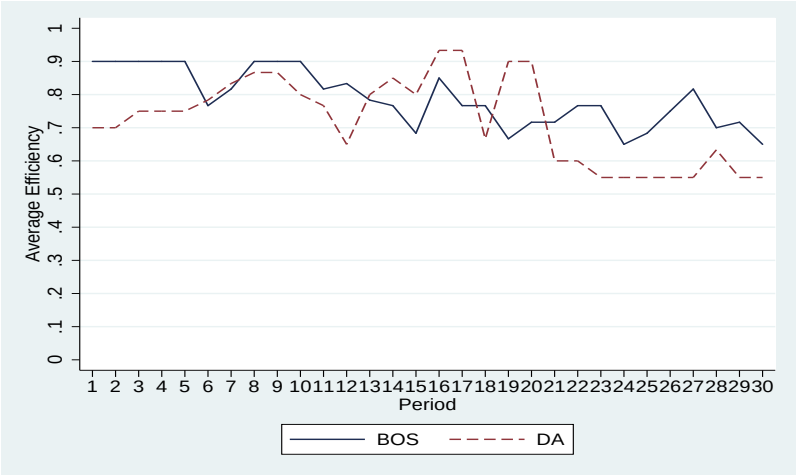
Complete, Unclear (CK2019's treatment)

(a) normalized efficiency, 4H2L



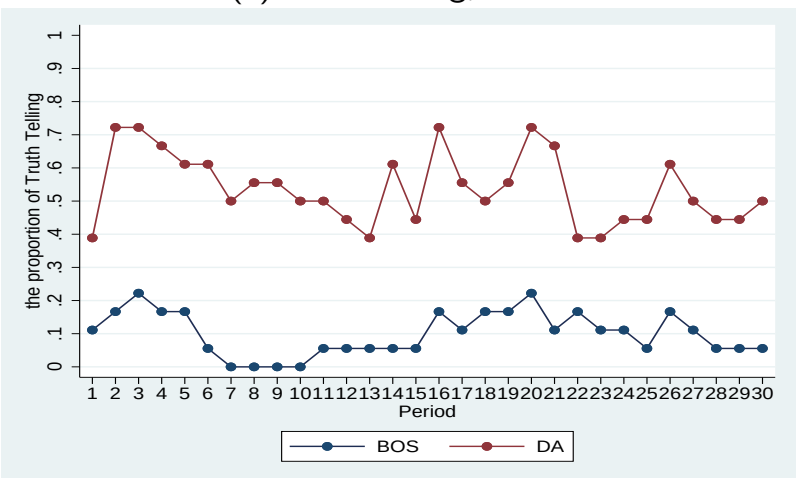
$p = 0.4403$ (BM) $p = 0.0092$ (KS)

(b) normalized efficiency, 3H3L



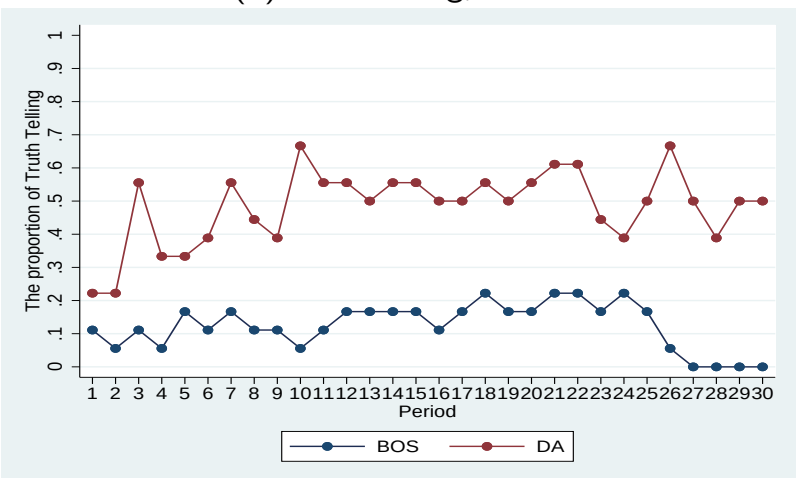
$p = 0.0063$ (BM) $p = 0.0011$ (KS)

(c) truth-tellling, 4H2L



$p < 0.0001$ (BM) $p < 0.0001$ (KS)

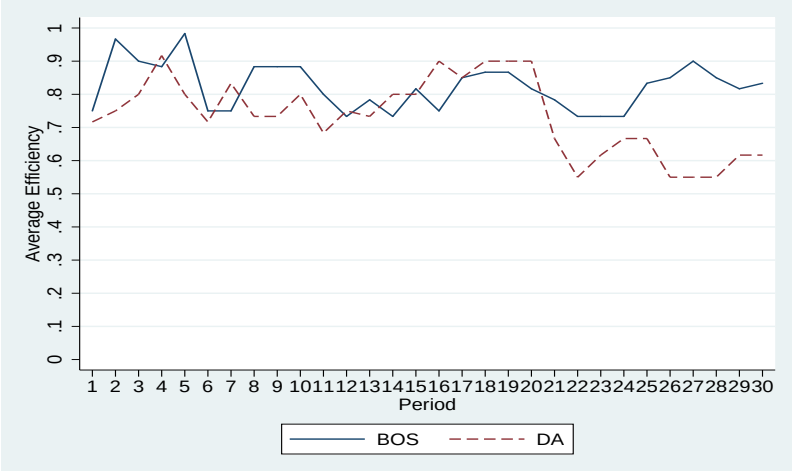
(d) truth-telling, 3H3L



$p < 0.0001$ (BM) $p < 0.0001$ (KS)

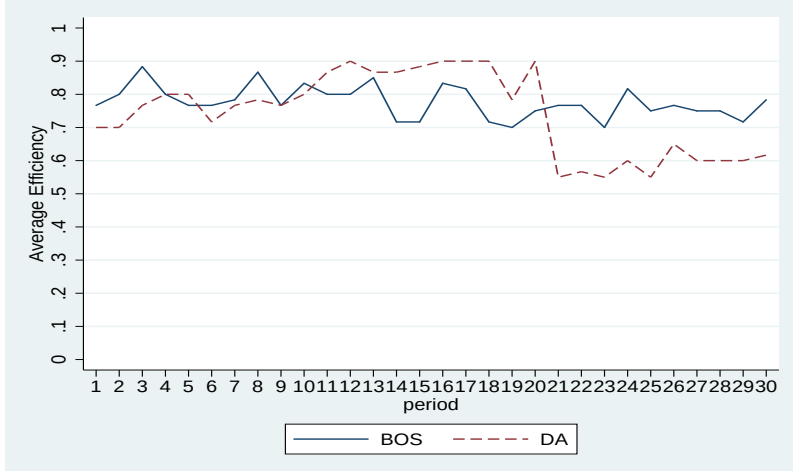
Complete, Unclear (CK2019's treatment)

(a) normalized efficiency, 2H4L



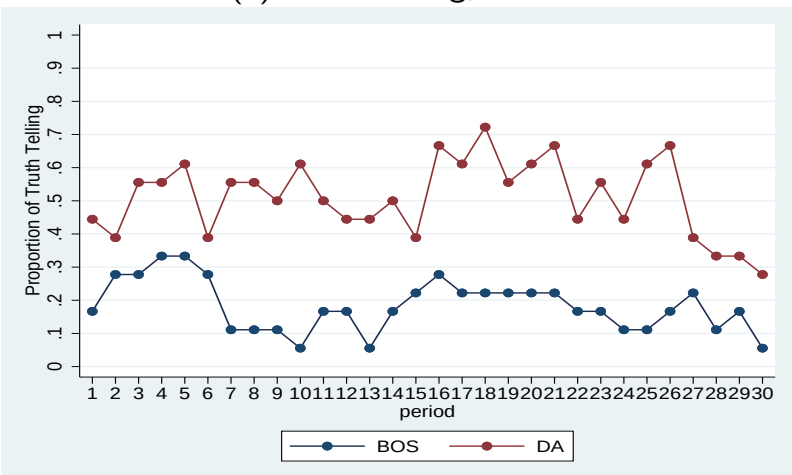
$p < 0.0001$ (BM) $p = 0.0033$ (KS)

(b) normalized efficiency, 6H



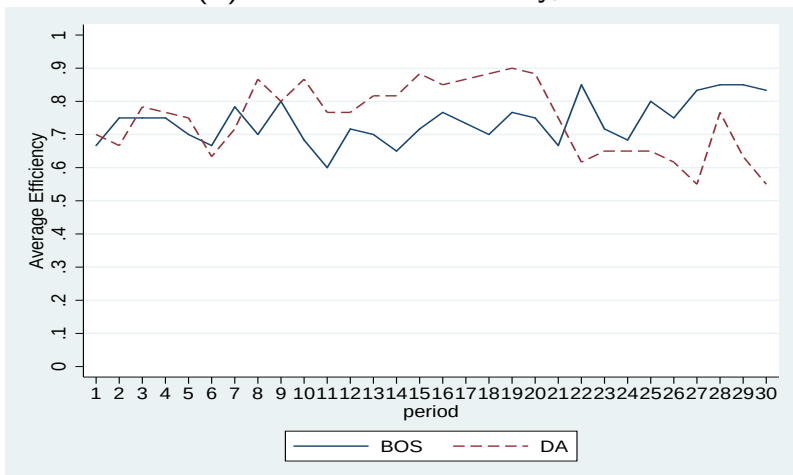
DA $\sim$ Bos relatively low in period 11-20 under Bos

(c) truth-tellling, 2H4L



$p < 0.0001$ (BM) $p < 0.0001$ (KS)

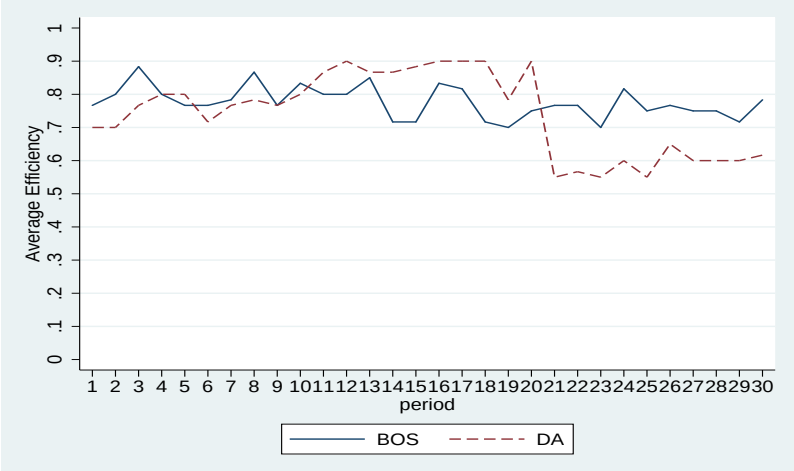
(d) normalized efficiency, 6L



DA $\sim$ Bos relatively low in period 11-20 under Bos

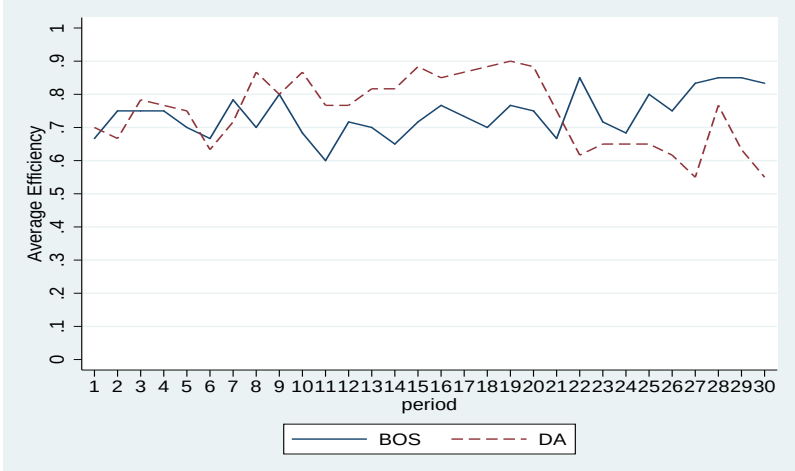
Complete, Unclear (CK2019's treatment)

(a) normalized efficiency, 6H



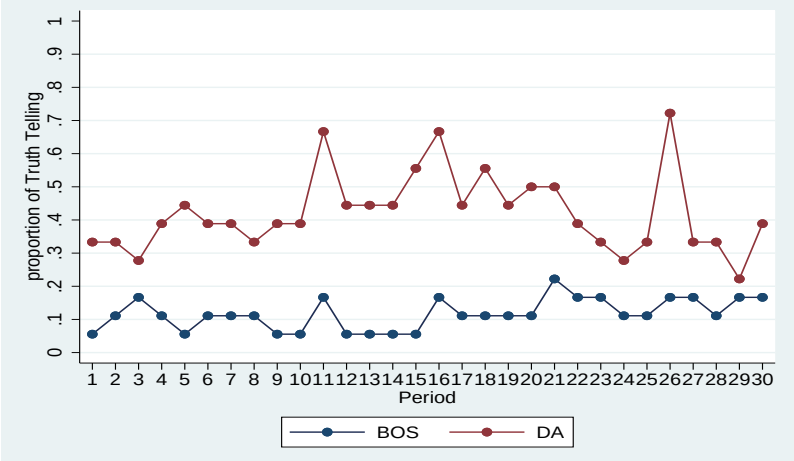
$p = 0.3373$ (BM) $p = 0.0149$ (KS)

(b) normalized efficiency, 6L



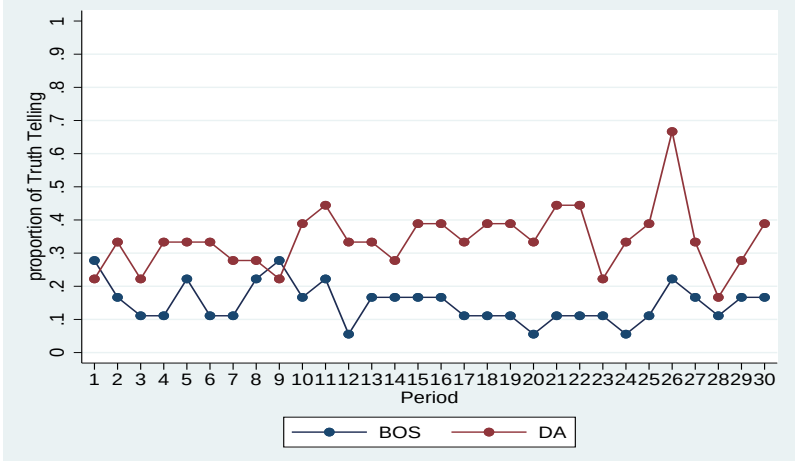
$p = 0.5735$ (BM) $p = 0.1163$ (KS)

(c) truth-tellling, 6H



$p < 0.0001$ (BM) $p < 0.0001$ (KS)

(d) truth-telling, 6L



$p < 0.0001$ (BM) $p < 0.0001$ (KS)

- We counted the # of observed frequency of TT with > 0.5 (> 0.2) under DA (Bos) in every 5 rounds.
  - There were no significant differences in the distribution of those observations between CK2019 and 2H4L. (differences observed in 3H3L)
- it was more plausible that we could replicate the same result as the one shown in CK2019 in the case of 2H4L.

Table: P-values for the KS test: (TT under DA and Bos)

	DA	> 0.5	Bos	> 0.2
round	CK2019	2H4L	CK2019	2H4L
$1-5$	4	3	0	1
$6-10$	4	3	0	0
$11-15$	1	0	0	0
$16-20$	1	5	1	1
$21-25$	2	3	3	0
$26-30$	1	1	0	0
p-value (KS)	$---$	0.8928	$---$	0.44131

4. Final Remarks

Subjects' cognitive ability scores and information affect allocative efficiency and subjects' behavior.

- Subjects' cognitive ability and information on school's priority orders over students matter in comparison of allocative efficiency between Bos and DA.
- As for sensitivity of behavior, their cognitive ability and information on their preferences over schools are important.

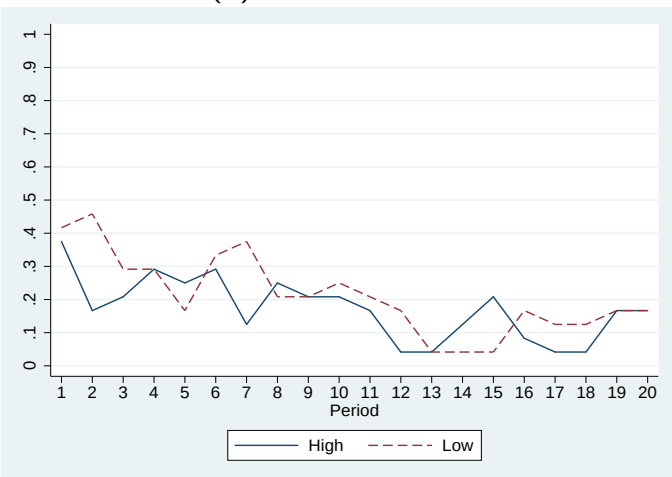
4. Final Remarks: More on Subjects' Behavior under DA

- In the case of 4 students and 4 schools, many subjects chose truth-telling strategy or district schools as the top choice; e.g., 89.6 % of observed choices made by high score holders in the CU treatment (CK2019)
- skipping down strategy (**Kawagoe et al. 2018, GEB**): stating preferences in order of schools that value them more highly
  - In this experiment with district schools, skipping down strategy = choosing district schools as the top choice.
- 3-school and 5-student, Complete and Clear info under plain DA (not DA with majority quota or with minority reserve): average 62.5-69.44% of preference statements were skipping-down strategy.

relative frequencies of skipping-down strategy observed in CK2019's treatment (Complete and Unclear info.)

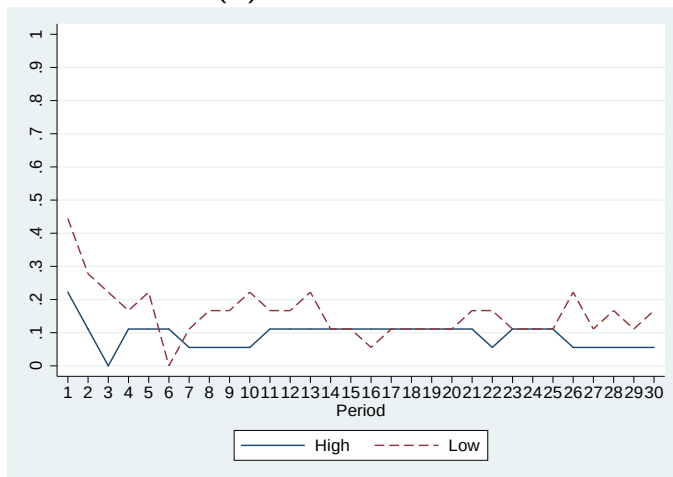
Complete, Unclear (CK2019)

(a) 4-schools, DA



high = 0.1729, low = 0.2125 p = 0.1199 (BM)

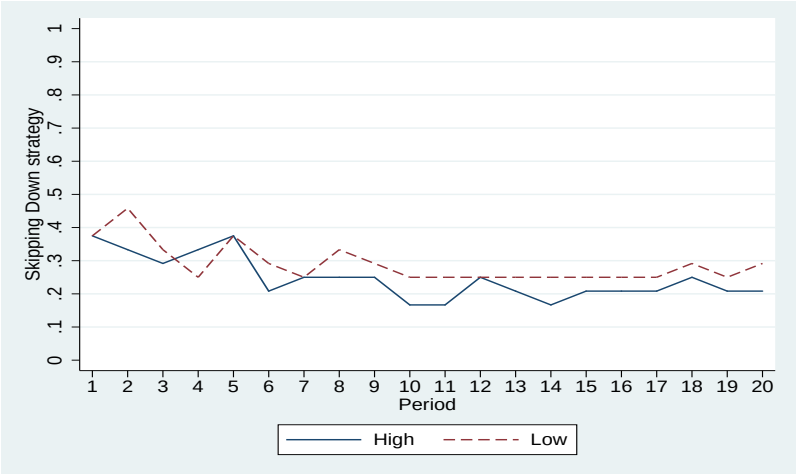
(b) 6-schools, DA



high = 0.0926, low = 0.1574 p = **0.0012** (BM)

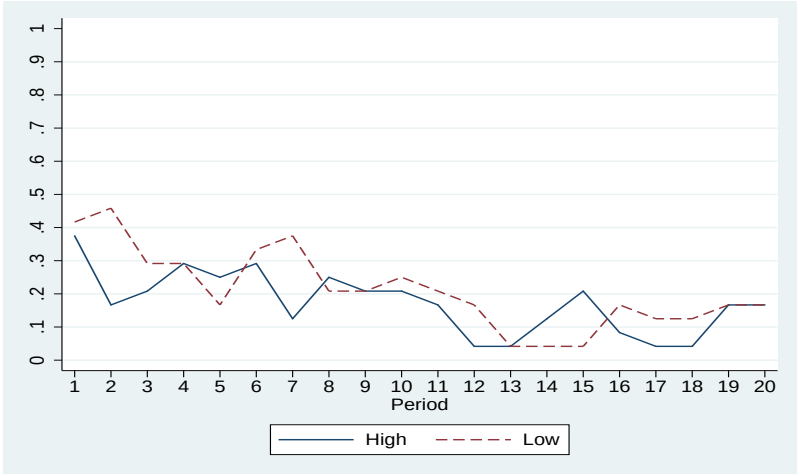
relative frequencies of skipping-down strategy observed in 4 schools and 4 students

(a) skipping-down, Complete, Clear



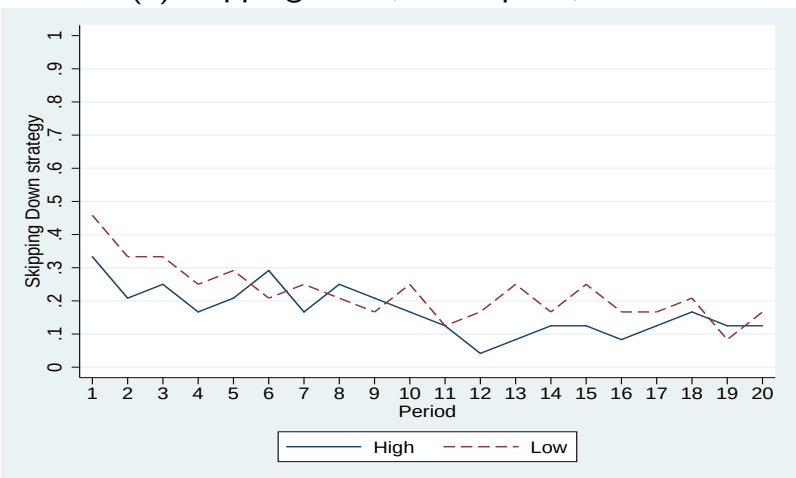
high = 0.2458, low = 0.2896 p = 0.1258 (BM)

(b) skipping-down, Complete, Unclear (CK2021)



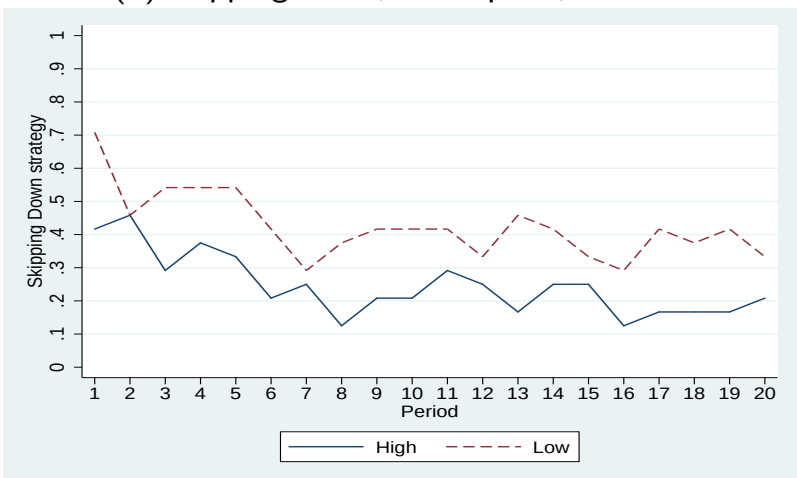
high = 0.1729, low = 0.2125 p = 0.1199 (BM)

(c) skipping-down, Incomplete, Clear



high = 0.1688, low = 0.2250 p = **0.0282** (BM)

(d) skipping-down, Incomplete, Unclear



high = 0.2458, low = 0.4250 p < **0.0001** (BM)

In the case of 4 schools and 4 students

- It is intuitively understandable that the relative frequencies of dropping-down strategy were higher for both high and low score holders in treatment with incomplete and unclear information than in the other treatment.
- The apparent diminishing tendency was observed in every treatment of information, regardless of groups of subjects in cognitive ability scores. ... learning?

4. Final Remarks: More on Subjects' Behavior under Bos

- sincere player and sophisticated player in **Pathak and Sönmez (2008, AER)**: players who represent their true preferences and those who play the Nash equilibrium strategy
  - In the case of 4 schools and 4 students, choices made by sophisticated or sincere players are, e.g., 95.8 % in the high score group in the CU treatment (CK2019)

Sorry! We are still struggling to find all the Nash equilibria in the environment of 6 schools and 6 students under Bos.
... They are too many...

4. Final Remarks: Software

Search with "Naoki Watanabe, Keio". At the bottom of my website there are some softwares:

Excel for Two-Sided Matching ver.3.2

http://labs.kbs.keio.ac.jp/naoki50lab/sample3_v2.xlsm

The User's Manual

http://labs.kbs.keio.ac.jp/naoki50lab/manualEn2Sh.pdf

A Brief Note on a Personnel Assignment in a Japanese Electrical Company $\cdots$ The full version is sold as a case material of Keio Business School; Case 91-18-3199

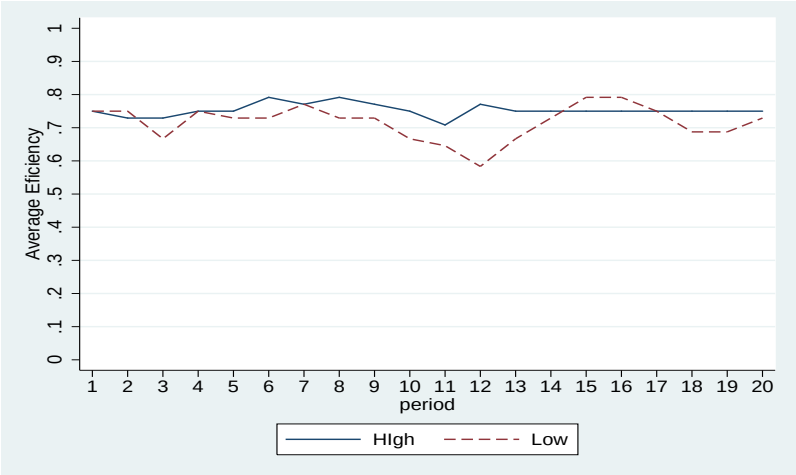
http://labs.kbs.keio.ac.jp/naoki50lab/personnel2EnSh2.pdf

Softwares for TTC and VCG are also there.

Appendix A. Experimental Results: An Overview

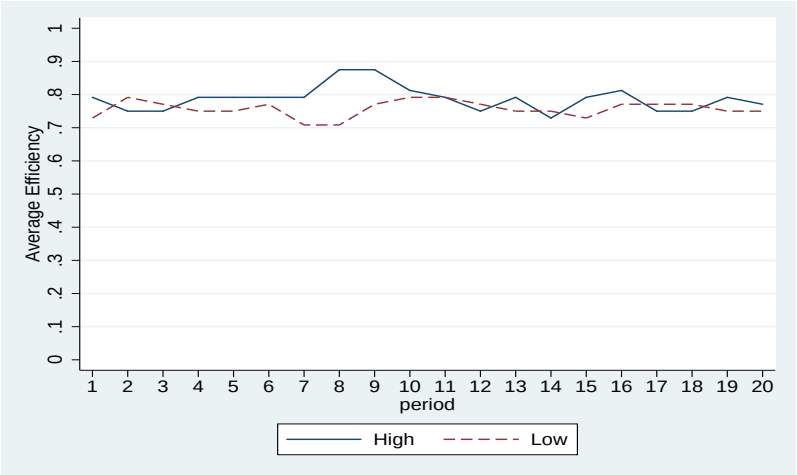
Complete, Unclear (CK2019's treatment)

(a) normalized efficiency, Bos



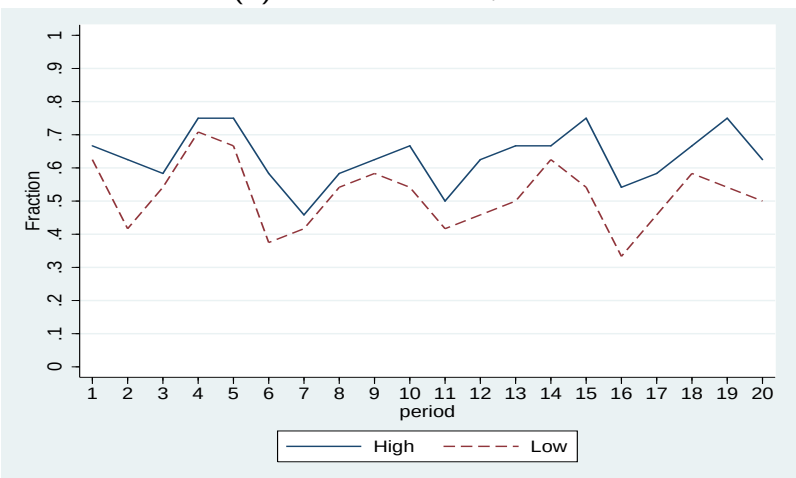
$p = \mathbf{0.0016}$ (BM) $p = \mathbf{0.0135}$ (KS)

(b) normalized efficiency, DA



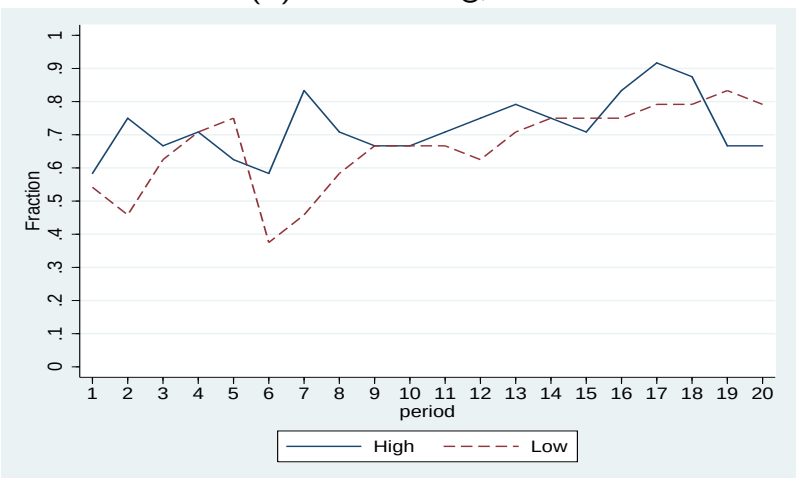
$p = \mathbf{0.0013}$ (BM) $p = <\mathbf{0.0001}$ (KS)

(c) district school, Bos



$p = <\mathbf{0.0001}$ (BM) $p = \mathbf{0.0047}$ (KS)

(d) truth-telling, DA



$p = 0.2720$ (BM) $p = 0.7937$ (KS)

Appendix B. Truth-telling under DA (CK2019)

	Bos (uncond)	DA (uncond)	Bos (Q1 correct)	DA (Q1 correct)
comp, unclear (Chen-Kesten2016)	$s = 480$	$s = 480$	$a = 460$ $b = 440$	$c = 340$ $d = 280$
high (H)	$156/s = 0.33$	$347/s = 0.72$	$153/a = 0.33$	$254/c = 0.75$
low (L)	$166/s = 0.35$	$319/s = 0.67$	$166/b = 0.38$	$208/d = 0.74$
p-value (Fisher)	0.5384	**0.0293**	0.1641	0.9264
incomp, unclear	$s = 480$	$s = 480$	$a = 440$ $b = 440$	$c = 280$ $d = 160$
high (H)	$178/s = 0.37$	$295/s = 0.62$	$155/a = 0.35$	$190/c = 0.68$
low (L)	$136/s = 0.28$	$211/s = 0.44$	$107/b = 0.24$	$86/d = 0.54$
p-value (Fisher)	**0.0048**	**<0.0001**	**0.0005**	**0.0041**

H_0: no difference in relative frequencies observed between H and L.
P-values for two-sided (one-sided) Fisher exact test (DA (uncond)).

Appendix B. District School as Top Choice under Bos (CK2019)

	Bos (uncond)	DA (uncond)	Bos (Q1 correct)	DA (Q1 correct)
comp, unclear (Chen-Kesten2016)	$s = 480$	$s = 480$	$a = 460$ $b = 440$	$c = 340$ $d = 280$
high	$304/s = 0.63$	$83/s = 0.17$	$288/a = 0.63$	$56/c = 0.17$
low	$249/s = 0.52$	$102/s = 0.21$	$255/b = 0.58$	$54/d = 0.19$
p-value (Fisher)	**0.0004**	0.0703	0.1728	0.3984
incomp, unclear	$s = 480$	$s = 480$	$a = 440$ $b = 440$	$c = 280$ $d = 160$
high	$286/s = 0.60$	$118/s = 0.25$	$269/a = 0.61$	$55/c = 0.20$
low	$326/s = 0.68$	$204/s = 0.43$	$315/b = 0.72$	$63/d = 0.39$
p-value (Fisher)	**0.0088**	**<0.0001**	**0.0013**	**<0.0001**

H_0: no difference in relative frequencies observed between H and L.
P-values for two-sided (one-sided) Fisher exact test (DA (uncond)).

Appendix B. Wrap-up

- We can remove the significant differences in subjects' behavior between two groups of cognitive ability observed in CK2019's treatment.
- Confine our analysis to the data taken from the subjects with correct answer to Q1.

Appendix C: DA algorithm

1. Each student has his or her preference over schools to which he or she is to be assigned, and each school has its priority orders for students. The preferences and priority orders are represented by ranking without ties.
2. Quota: the maximal number of students that can be accepted by each school, i.e., capacity.

An assignment of students to schools is called a matching and denoted by μ, such as

$$\mu = \begin{pmatrix} a & b & c & d & e & f \\ X & Z & Z & X & Y & Z \end{pmatrix},$$

where, e.g., students a and d are assigned to school X.

- A matching is deemed **unstable** if it sends a student to a school when there is another school that is preferred by the student, and either has room for him or her or could make room by rejecting someone else it prefers less.
- For each two-sided matching problem, in general, there are some stable matchings.
- Stability implies **(Pareto) efficiency for both sides**.

Example

Table: Students' preferences over schools

a:	X	Y	Z	d:	Y	Z	X
b:	Z	Y	X	e:	Y	Z	X
c:	Z	Y	X	f:	Z	X	Y

Table: Schools' priority orders for students

(2)	X:	b	c	a	d	f	e
(1)	Y:	a	d	c	e	f	b
(3)	Z:	f	e	c	d	b	a

A stable matching

$$\mu^* = \begin{pmatrix} a & b & c & d & e & f \\ X & X & Z & Y & Z & Z \end{pmatrix}$$

is computed by the student-proposing DA algorithm as follows.

Table: Computation process of μ^* by the student-proposing DA algorithm

		round 1	round 2	round 3	round 4
(2)	X	a	a	a	a, b
(1)	Y	d, ~~e~~	d	d, ~~b~~	d
(3)	Z	b, c, f	~~b~~, c, f e	c, f, e	c, f, e

The student-proposing DA algorithm guarantees the following properties.

- **Strategy-proofness for students**: For each student, whatever other students report their preferences over schools, it is never assigned to the more preferable school for him or her by misrepresenting their true preference.

The next property is, unfortunately, not guaranteed at the stable matching computed by the student-proposing DA algorithm.

- **(Pareto) efficiency among students**: In order to reassign a student to a higher ranked school, there is at least one student who must be transferred to a lower ranked school for him or her.

- Deferred acceptance (replaceability of applications) induces students to tell their true preferences and guarantees efficiency for both sides but sometimes fail in attaining efficiency among students.
- In fact, CK2019 reported that there was no significant difference in efficiency between Bos and DA in the case of 4 schools and 4 students and that Bos is better in terms of efficiency (measured by an numerical index) than DA in the case of 6 schools and 6 students.