An Experimental Study of an Approximate DGS Mechanism: Price Increment, Allocative Efficiency, and Seller's Revenue

Y. Izunaga[1], S. Takahashi[2], N. Watanabe[3]

[1]Kyusyu U., [2]Electro-Com. U., [3]Keio U.

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1. Introduction: Research Questions

Question: To find an appropriate increment of prices under an approximate DGS algorithm in simultaneous ascending auctions with bidders under the unit-demand constraint

- larger increment of prices ⇒ (1) allocative efficiency and seller's revenue? (2) subjects' behavior? (3) satisfiability of DGS boundary formula (approximation error)?
  - Vickrey-Clarke-Groves (VCG) mechanism: difficult to understand, one-shot bids
  - Demange-Gale-Sotomayor (DGS) algorithm: easy to understand, multi-stage bids ⇒ saving time by large increments of prices
- an approx DGS: bidding one by one in turn
  ⇒ bidders can take their own time for decision
  ⇒ more important to save time for practical use: secured real property auctions conducted by the Japanese courts

1. Introduction: Key Concepts

Experiment

- 2 ($= m$) items are auctioned off to 3 ($= n$) bidders.
- Increments of prices are 50, 20, 10 ($= d$).
- Every bidder can place a bid for 1 item.
- Time limit for placing a bid is 10 sec.
- Valuations are independently distributed between 0 and 500.

Key Concepts

- sincere bidding: bidding on the item the difference between the valuation and the offered price of which is larger
- DGS bounds (DGS, 1986): under approx DGS, the difference between the price settled by sincere bidders and the minimum price at the Walrasian equilibria should be smaller than $d \cdot \min(n, m) = 2d$.

1. Introduction: Main Results

(1) efficiency rates = 95% ($d = 50$), 98% ($d = 20, 10$)

(2) seller's revenue = 99% ($d = 50$), 104% ($d = 20, 10$),
compared to the case under VCG

(3) When $d = 50$, significantly more cases of overbids and underbids ⇒ large standard deviation in the case of $d = 50$

(4) max $t/T = 3.0$ ($d = 50$), 7.0 ($d = 20$), 13.5 ($d = 10$)

- $T =$ the computed number of bids assuming that all bidders place sincere bids
- $t =$ the observed number of bids that are placed by subjects

(5) frequency of sincere bids increased as t/T increased.

(6) DGS boundary formula was satisfied at the rates of 85% ($d = 50$) and 71% ($d = 20, 10$).

- When $d = 10$, it was satisfied at 100% for item 1, but the rate decreased to 71% for item 2. (visual position for items on subjects' monitors?)

Ans.: We recommend $d = 20$ as an appropriate price increment.

2. Experimental Design: Session Info

- venue: ISER, Osaka University
- participants: undergraduate students
- 5 groups of 3 bidders, randomly matched in every auction.
- 2 practice rounds + 13 rounds for 2 virtual items
- points subjects earned were selected by the computer randomly from 4 out of the 13 rounds
  - The total amount of payment was total points for those 4 rounds in JPY (1 point = 1 JPY) plus a compensation of 1,800 JPY for their participation.
- Data obtained in the last 10 auctions are used for our analysis.
- average Raven score of subjects = about 12.5 for 16 questions excerpted from the 48 questions of the Raven APM test (measuring subjects' ability of pattern recognition).

2. Experimental Design: Raven Score of Non-students

The Raven scores of non-student general public were collected at the northern Osaka prefecture.

	Num. of Obs.	Mean	Std. Dev.	Min.	Max.
2016.4-2018.3	934	7.935	3.283	0	16

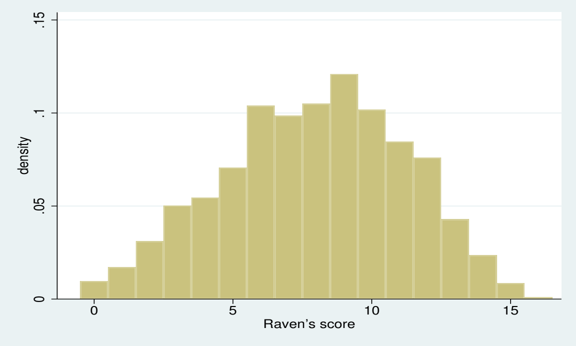

Table: Features of the experimental sessions.

session no.	amount of increment	# of subj.	session date	avg. payment per subject
1	50	15	Apr. 23, 2018	2156.47
2	50	15	Apr. 23, 2018	2168.27
3	50	15	Apr. 24, 2018	2052.60
4	50	15	Apr. 24, 2018	2227.60
5	20	15	Oct. 26, 2018	2118.00
6	20	15	Oct. 26, 2018	2266.67
7	20	15	Nov. 30, 2018	2170.13
8	20	15	Nov. 30, 2018	2213.27
9	10	15	Mar. 19, 2019	2214.87
10	10	15	Mar. 19, 2019	2197.87
11	10	15	Mar. 20, 2019	2293.20
12	10	15	Mar. 20, 2019	2230.13
13	sim. 1st price	15	Nov. 4, 2020	2063.87
14	sim. 1st price	15	Nov. 4, 2020	2056.33

2. Experimental design: approx DGS

- valuation of each item $\sim U[0, 500]$, independently of the other bidders' valuations. private information.
- Each bidder placed a bid or withdraw in turn, corresponding to the offered prices of the items going up from 0 in pre-announced increments of $d = 50$ $(20, 10)$.
- Bidding was subject to a time limit of 10 seconds.
  - Unless any bids were not placed within the time limit, the bidder was automatically withdrawn by the computer.

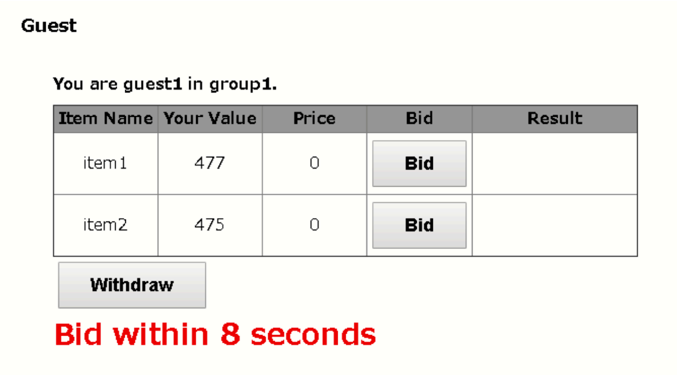

Figure: Guest 1 can choose to either bid on item 1, bid on item 2, or withdraw, by pressing on the corresponding button.

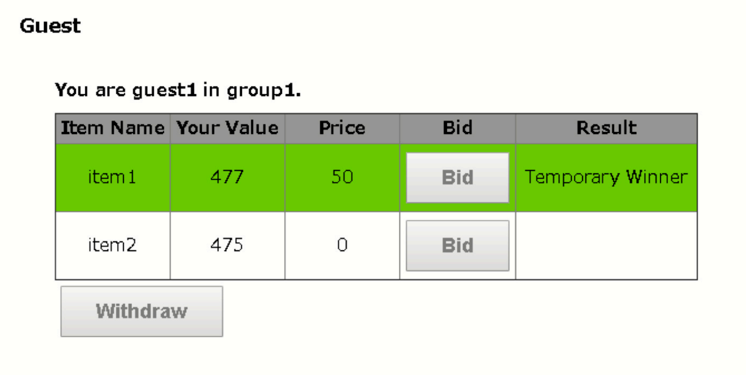

Figure: Guest 1 is the tentative winner of item 1. The price of item 1 is raised up to 50 for the others.

- tie-breaking rule: If Guest 2 bids on item 1, then Guest 2 becomes the new tentative winner.
- After Guest 3 made his or her first decision, if any of the bidders withdraws, then the auction ends.

- If no bidder withdraws, then the auction proceeds.
- In the next step and after, the guest who got out of being a tentative winner earlier makes a choice in the same way.
- If any of the bidders withdraws, then the auction ends.

Figure: Guest 1 obtains item 1 at the price of 200. Note that 250 the displayed on the monitor was the price for the other bidders.

3. Results: Efficiency Rate and Seller's Revenue

We analyze the data taken from the last 10 out of 13 rounds. Let $\hat{\boldsymbol{x}} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ be an observed allocation. The rate of efficiency is defined by

$$\frac{\sum_{i \in \mathcal{N}} V_i(\hat{x}_i)}{\text{the optimal value of the VCG mechanism}}. \tag{1}$$

The rate of seller's revenue (profit) is defined by

$$\frac{\text{the total amount of observed payments}}{\text{the total amount of optimal payments}}, \tag{2}$$

where the total amount of optimal payments is calculated under the assumption that every bidder truthfully bids his or her (unit) valuations under the VCG with unit-demand constrained bidders.

Hypothesis 1: As the increment of prices are larger, (1) the rate of efficiency decreases on average, and (2) the rate of seller's revenue decreases on average.

Observation 1: The Hypothesis 1 was affirmatively confirmed between the case of $d = 20$ and $d = 50$.

- Sample size = 200 (10 rounds, 5 groups, 4 sessions) for each price increment.
- The two-sided Brunner-Munzel test (BM)
- the null hypothesis is that there is no difference in those averages between two different price increments, which is rejected at the 5% significance level.

Table: The rates of efficiency.

<table>
<tr><td>price incre.</td><td>d=10</td><td>d=20</td><td>d=50</td></tr>
<tr><td>mean</td><td>0.9762</td><td>0.9675</td><td>0.9574</td></tr>
<tr><td>st.dev.</td><td>0.0773</td><td>0.0953</td><td>0.1044</td></tr>
<tr><td>p-value (BM)</td><td colspan="2">0.1202</td><td></td></tr>
<tr><td></td><td></td><td colspan="2">0.0323</td></tr>
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Table: The rates seller's revenue.

price incre.	d=10	d=20	d=50
mean	1.0433	1.0353	0.9997
st.dev.	0.2496	0.3043	0.6230
p-value (BM)	0.6792		
		0.0024	

Overbids were observed. (15x10=150 obs. in a session)

- $d = 50$: 29, 20, 23, and 30 cases; $d = 20$: 11, 11, 14, and 7 cases; $d = 10$: 6, 4, 10, and 5 cases
- Not necessarily in early rounds. $\Rightarrow$ Non-pecuniary benefits?
- In taking the average, the amount of underbids cancelled the magnitude of overbids in $d = 50$. (The large increment of prices incurred a larger st. dev. in seller's revenue.)

3. Results: Subjects' behavior

Hypothesis 2: Subjects tend to bid more sincerely as auctions proceed.

Observation2: Affirmatively confirmed.

- T: the desirable number of bids in an auction in which every bidder sincerely bids for any turn. (derived by simulation)
- t: the observed number of bids in the auction.

$$\frac{t}{T} \neq 1 \Rightarrow \text{Some bidders did not sincerely bid.}$$

The label ℓ_t indicates whether the player $b(t)$ bids sincerely according to his or her valuation for each item, that is,

$$\ell_t = \begin{cases} 0 & \text{if } \operatorname{argmax}\{\, v_i^{b(t)} - p_i^t \mid i \in I \,\} = i(t), \\ 1 & \text{otherwise. (insincere bids)} \end{cases} \tag{3}$$

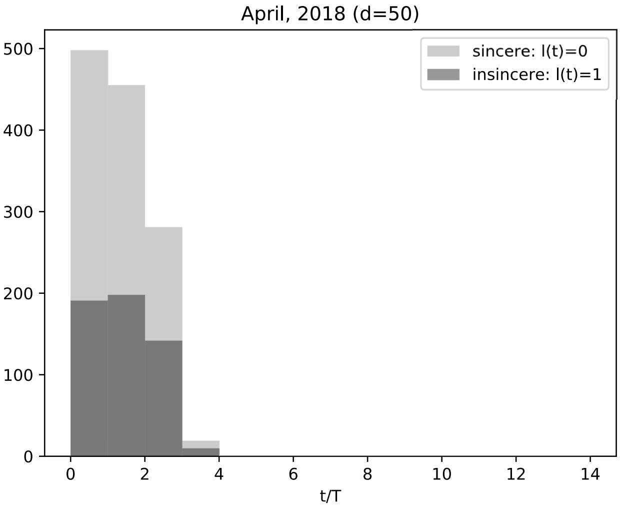

Figure: Price increment = 50.

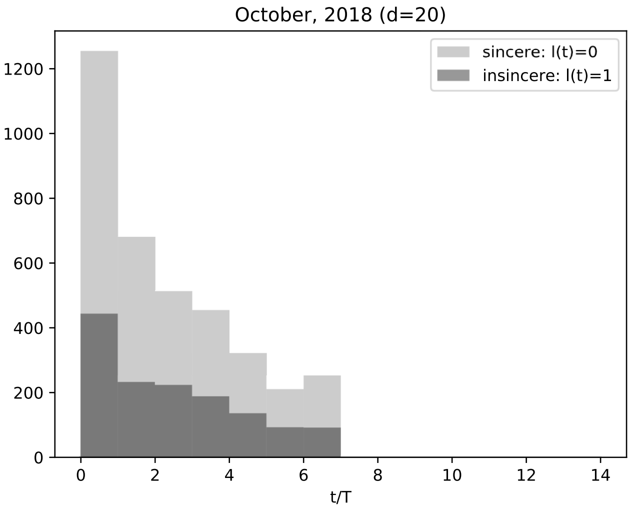

Figure: Price increment = 20.

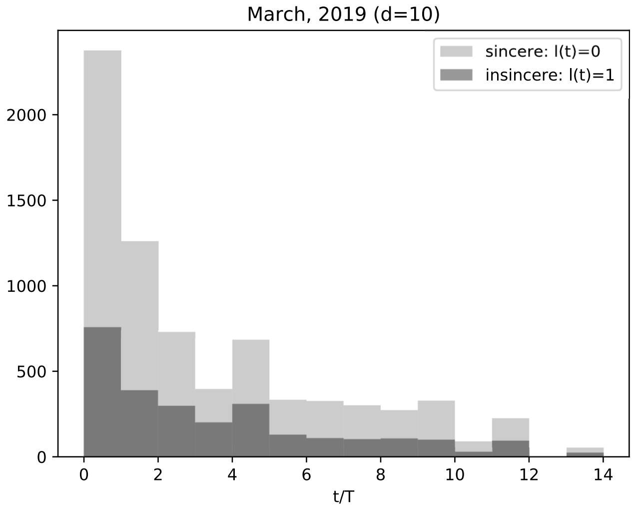

Figure: Price increment = 10.

(Those histograms does not represent the ratio of 0 and 1. They are imply overlapped.)

$$
\begin{align}
x_{t,1} &= t/T, \tag{4}\\
x_{t,2} &= v_1^{b(t)} - p_1^t, \text{(valuation - price, item 1)} \tag{5}\\
x_{t,3} &= v_2^{b(t)} - p_2^t, \text{(valuation - price, item 2)} \tag{6}\\
x_{t,4} &= \begin{cases} 0 & \text{if } i(t) = i^{\text{pre}}(t), \\ 1 & \text{otherwise, (bidder } i \text{ changed options)} \end{cases} \tag{7}
\end{align}
$$

where $b(t)$ stands for the bidder at time t, $i(t)$ and $i^{\text{pre}}(t)$ represent the item for which $b(t)$ bids and the item for which $b(t)$ bids at the most recent time, respectively. ($x_{t,4} = 0$ for $t \leq 3$.)

Table: Summaries of dataset and classification by SVM-SM.

price incre.	# of samples	estimates w_1	w_2	w_3	w_4	misclass ratio
50	1794	-0.6000	-0.0006	-0.0003	0.0039	0.7648
20	5091	-0.2385	-0.0007	-0.0013	0.1240	0.7714
10	10019	-0.1238	-0.0017	-0.0016	0.1851	0.7544

- Misclassification ratios were remarkably high in discriminant analysis.

Table: Results of Logistic Regression Analysis.

price incre.		estimates w_1	w_2	w_3	w_4	Pseudo R^2
50		-0.5441	-0.0001	0.0006	0.1915	-0.0350
	p-val	$< \mathbf{0.0001}$	0.8885	0.0856	0.0571	
20		-0.2689	0.0004	-0.0001	0.0356	-0.0685
	p-val	$< \mathbf{0.0001}$	**0.0379**	0.6200	0.5857	
10		-0.1305	-0.0011	-0.0009	0.3757	-0.0830
	p-val	$< \mathbf{0.0001}$	$< \mathbf{0.0001}$	$< \mathbf{0.0001}$	$< \mathbf{0.0001}$	

- As t/T increases, the fraction of insincere bids decreases in $d = 50, 20, 10$. $\Rightarrow$ How long did the chain of sincere bids last immediately before the settlement of the auctions?
- When $d = 10$, bidders who changed options did not place sincere bids.
  - We observed this tendency also in the cases of $d = 20, 10$ by using another statistical test.

- When the bid immediately before the settlement (last bid) was a sincere bid, how long did the chain of sincere bids last, counting backwards from the last bid?
  - It was only 1.315 on average, when $d = 50$.

We are currently counting the number of chains of sincere bids which last at least 3 immediately before the last bids.

3. Results: Approximation Error in DGS Boundary Formula

The boundary of the outcomes under Approx DGS:

$$|p_i - p_i^*| \leq d \cdot \min\{n, m\},$$

where p_i is the final price of item i the winner pays under approx DGS, and p_i^* represents the minimum price of item i at Walrasian equilibria which attained by exact DGS.

Hypothesis 3: As increment of prices becomes larger, the fraction of approximation errors in the DGS boundary formula increases.

Observation 3: Affirmatively observed between $d = 50$ and $d = 10$: It was satisfied with the cases of 71% $(d = 10, 20)$ to 85% $(d = 50)$ on average.

Table: Summaries of Results: Approximation Formula.

d	$\lvert p_1 - p_1^* \rvert < 2d$	$\lvert p_2 - p_2^* \rvert < 2d$	both	$p_1 > p_1^*$	$p_2 > p_2^*$	$p_1 < p_1^*$	$p_2 < p_2^*$
50	0.915	0.905	0.850	0.435	0.440	0.560	0.560
20	0.815	0.815	0.710	0.490	0.515	0.490	0.455
10	1.000	0.710	0.710	0.480	0.520	0.470	0.445

- In the case of $d = 10$, the satisfaction rate dropped to 71% for item 2, although it was 100% for item 1.
- That large drop was not observed in the cases of $d = 50, 20$. The visual position for items on subjects' monitors affects bidding behavior? (ref. Takahashi et al. (2019) for the case of multi-unit auctions)

4. Concluding Remarks: Increment of Prices

- Large increment ($d = 50$): significantly many overbids and underbids $\Rightarrow$ large standard error in seller's revenue
- Small increment ($d = 10$): satisfaction rates of the DGS boundary formula are different between items (confusion?)

We recommend $d = 20$.

- No significant difference in allocative efficiency and seller's revenue in comparison with the case of $d = 10$. (Obs. 1)
- Time elapsed to settle auctions was, on average, shorter than that in the case of $d = 10$. (Obs. 2+) (We skipped showing statistical results in this presentation.)
- Satisfaction rates of DGS boundary formula were satisfied equally likely between items (Obs. 3)

4. Conclusions: vs. Simultaneous-Bid First-Price Auction

Simultaneous-bid first-price auctions are currently being used in secured real property auctions in Japan.

Under unit-demand constraint, we observed

- efficiency rate: mean $= 0.8241$, and st.dev. $= 0.2067$.
- seller's revenue: mean $= 1.3765$ and st.dev. $= 1.4302$.

Under approx DGS, the standard deviation in seller's revenue ranged from 0.24 to 0.62.
$\Rightarrow$ more difficult to predict the auction outcomes under simultaneous-bid first-price Auction

In our sessions, demand reduction was not observed even once among 130 auctions. (We did not expect this result.)