von Neumann-Morgenstern Stable Sets of a Patent Licensing Game: The Existence and Non-Existence

Toshiyuki Hirai Naoki Watanabe

Toyama, Econ Keio, Business Admin

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1. What is done

- The existence proof of von Neumann-Morgenstern stable sets in a game with a coalition structure.

  - The core is empty for almost every coalition structure, in the context of patent licensing.

    $\Rightarrow$ more difficult to show the existence.

  - Find a type of stable sets.

    $\Rightarrow$ A reduced game plays an important role.

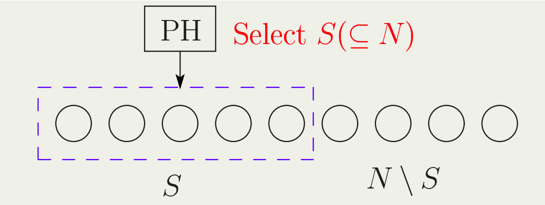
1. The outline of this talk

1. Introduction
2. The Model: Patent Licensing Game
3. Definitions: Solution Concept
4. The Analysis and Results
5. Symmetric Restrictions
6. An Implication to Producers' Surplus

2. Patent Licensing Game: stage (i)

- $N = \{1, \ldots, n\}$: set of symmetric firms ($2 \leq n < \infty$)
- player 0: (external) patent holder ($\{0\} \cup N$: set of players)
- 3-stage game

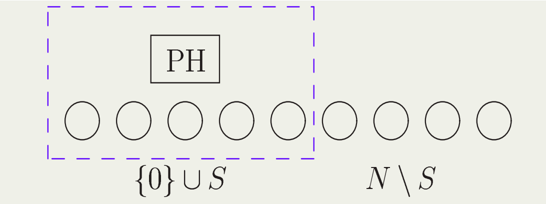
stage (i): A patent holder selects a set S of firms for license negotiations.



- Coalition $\{0\} \cup S$ forms only for license negotiation.
- $P^S = \{\{0\} \cup S, \{\{i\}\}_{i \in N \setminus S}\}$: permissible coalition structure

stage (ii): Firms in S negotiate license fees with the patent holder and make payments.

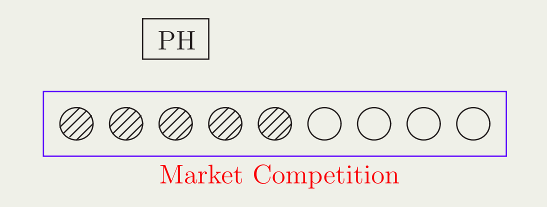
Negotiation on Payments



- Analyze the negotiation for each coalition structure P^S, assuming that all firms in S are given a license for simplicity.
- Check the acceptance of payments by each firm after finding the bargaining outcome.

2. Patent Licensing Game: stage (iii)

stage (iii): Knowing that which firms are licensed, each firm in N competes in the market. (Any cartels are prohibited.)



- When t firms are licensed, each licensee obtains the equilibrium gross profit $W(t)$ and each non-licensee who uses an old technology obtains the equilibrium gross profit $L(t)$.
- Assume that $W(t) > L(0) > L(t)\ \forall t = 1, \ldots, n-1, (n)$.
  Negative eternality arises in L(s)
- Each firm accepts the payment if it is $L(s-1)$ or more.

2. Remarks

- No negotiation process is specified at stage (ii).
  - The patent holder might negotiate with each firm in S on a one-by-one basis in dynamic bargaining procedures.

    ⋯ Aumann-Maschler's view on cooperative games

- We analyze a more serious case where the patent holder does not have any production facility ("external" patent holder); The core is empty in almost all cases.
  - Wako (2010, Algorithmica) proved that every marriage game has a unique stable set ($=$ core). $\Leftarrow$ Ehlers (2007, JET)
  - Núñez and Rafels (2013, JET) showed the existence of stable sets ($=$ core) in assignment games. $\Leftarrow$ Núñez and Rafels (2002, IJGT; 2009, GEB) $\Leftarrow$ Solymosi and Raghavan (2001, IJGT)

2. Major questions in the literature

- How many firms should be licensed? How much can the patent holder obtain?
    - Kamien and Tauman (1985, JE; 1986, QJE)
      take-it-or-leave-it offer in stage (ii).
        - Katz and Shapiro (1985, Rand; 1986, QJE);
          Add one more stage for duopolists' investments in technology development before stage (i).
        - Market structures are specified in noncooperative models;
          Muto (1993, GEB), Sen and Tauman (2007, GEB), etc.
    - Watanabe and Muto (2008, IJGT)
      license agreements as negotiation outcomes in stage (ii)
        - A genaral model.
        - Tauman and Watanabe (2007, ET) allows cartels.

3. More specifications: Assumptions

- **(I) Well-orderedness**
  $W(1) > W(2) > \cdots > W(n) > L(0) > L(1) \geq \cdots \geq L(n-1)$.

- **(II) Nonincreasing difference in L**
  $L(k-1) - L(k)$ is nonincreasing in $k = 1, ..., n-1$.

- **(III) Nonincreasing difference in W**
  $W(k) - W(k+1)$ is nonincreasing in $k = 1, ..., n-1$.

  - (III) is assumed only for Propositions 3 and 4.

- All assumptions are satisfied in the linear Cournot markets in stage (iii).

3. A bargaining game in stage (ii)

$(\{0\} \cup N, v, P^S)$: a game with a coalition structure

$\cdots$ Aumann and Drèze (1974, IJGT)

- $v : 2^{\{0\} \cup N} \to \mathbb{R}$; characteristic function
  - $v(\{0\}) = v(\emptyset) = 0$.
  - $v(\{0\} \cup T) = tW(t)$ for all nonempty $T \subset N$.
  - $v(T) = tL(n-t)$ for all nonempty $T \subset N$.

$v(T) = tL(\rho(t))$, where $L(\rho(t)) = \min_{r=|R|:R \subseteq N \setminus T} L(r)$. The external patent holder gains nothing without licensing, firms in $\{0\} \cup T$ use the patented technology, whereas the other firms use the old technology.

I^S: set of imputations under P^S, where

$$I^S = \left\{ \begin{array}{c} x = (x_0, x_1, \ldots, x_n) \\ \in \mathbb{R}^{n+1} \end{array} \;\middle|\; \begin{array}{l} x_0 + \sum_{i \in S} x_i = sW(s), \\ x_0 \geq v(\{0\}) = 0, \\ x_i \geq v(\{i\}) = L(n-1) \text{ for all } i \in S, \\ x_i = L(s) \text{ for all } i \in N \setminus S \end{array} \right\}$$

3. Dominance relation

Dominance relation

Let $x, x' \in I^S$.

We say that x dominates x' via $T \subset \{0\} \cup N$, denoted by $x \succ_T x'$, iff

- $T \cap (\{0\} \cup S) \neq \emptyset$,
- $\sum_{i \in T} x_i \leq v(T)$,
- $x_i > x'_i$ for all $i \in T \cap (\{0\} \cup S)$.

We say that x dominates x', denoted by $x \succ x'$, iff x dominates x' via some $T \subset \{0\} \cup N$.

Stable set

$K^S \subset I^S$ is a stable set for a bargaining game $(\{0\} \cup N, v, P^S)$ if K^S satisfies the following conditions.

Internal stability: For any $x, x' \in K^S$, $x \succ x'$ does not hold.

External stability: For any $x' \in I^S \setminus K^S$, there exists some $x \in K^S$ such that $x \succ x'$.

- For any z $(0 \leq z \leq sW(s))$, define $H^S(z) = \{x \in I^S | \ x_0 = z\}$.
- Since we are interested in the PH's revenue, we concentrate on a stable set K^S such that $K^S \subset H^S(z)$ for some z.

- The core C^S for a bargaining game $(\{0\} \cup N, v, P^S)$ is defined as

$$C^S = \{x \in I^S | \nexists y \in I^S, y \succ x\}.$$

Theorem 1

For any non-empty $S \subset N$, if $S \neq N$, then $C^S = \emptyset$. $C^N \neq \emptyset$ if and only if $n \in \arg\max_{s=1,\dots,n} s(W(s) - L(0))$.

4. Analysis 2: Reduced game

Given $(\{0\} \cup N, v, P^S)$ and z $(0 \leq z \leq sW(s))$, let $(S, \bar{v}_z^S)$ be a "reduced" game s.t.

$$v_x^S(T) = \begin{cases} 0 & \text{if } T = \emptyset \\ sW(s) - x_0 & \text{if } T = S; \\ \max_{r=0,\ldots,n-s}((t+r)L(n-t-r) - rL(s)) & \text{if } T \subset S \end{cases}$$

Lemma 1

For all $s = 1, \ldots, n$ and $t = 1, ..., s-1$,

$$v_x^S(T) = (t + n - s)L(s - t) - (n - s)L(s) \text{ if } T \subset S.$$

- The core $C(v_x^S)$ of the reduced game (S, v_x^S) is large if and only if for any non-empty $T \subset S$, there exists some $z \in C(v_x^S)$ such that $\sum_{i \in T} z_i \leq v_x^S(T)$.

Lemma 2

Let $S \subset N$ be non-empty and $x \in I^S$ be such that

$$sW(s) + (n - s)L(s) - nL(0) \leq x_0.$$

Assume that $C(v_x^S)$ is large. Let

$$K^S = \{x_0\} \times C(v_x^S) \times \{(L(s), \ldots, L(s))\}.$$

Then, for any $z \in I^S \setminus K^S$ such that $x_0 \leq z_0$, there exists some $y \in K^S$ such that $y \succ z$.

By Lemmas 1 and 2, we have the following key lemma.

Lemma 3

Let $S \subset N$ be non-empty and $x \in I^S$ satisfy

$$sW(s) + (n-s)L(s) - nL(0) \leq x_0 \leq \bar{s}(W(\bar{s}) - L(s)),$$

where $\bar{s} \in \arg\max_{t=0,\ldots,n-s} t(W(t) - L(s))$. Assume that $C(v_x^S)$ is large. Then,

$$\{x_0\} \times C(v_x^S) \times \{(L(s), \ldots, L(s))\}$$

is a stable set for $(\{0\} \cup N, v, P^S)$.

- Every convex game has the large core. (Sharkey, 1982)

Lemma 4

Let $S \subset N$ be non-empty and $x \in I^S$. Then,

(a) (S, v_x^S) is convex if

$$x_0 \leq A = sW(s) - 2(n-1)L(1) + (n-2)L(2) + (n-s)L(s).$$

(b) $sW(s) + (n-s)L(s) - nL(0) < A$.

- Set $x_0 = sW(s) - (n-s)L(s) - nL(0)$. Then, $C(v_x^S)$ is large.

By Lemmas 3 and 4, we have the following main theorem.

Theorem 2

Let $S \neq N$ be non-empty. If

$$sW(s) + (n - s)L(s) - nL(0) \leq \bar{s}(W(\bar{s}) - L(s)), \qquad (1)$$

where $\bar{s} \in \arg\max_{t=0,\dots,n-s} t(W(t) - L(s))$, then there exists a stable set K^S for $(\{0\} \cup N, v, P^S)$ such that $x_0 = sW(s) + (n - s)L(s) - nL(0)$ for any $x \in K^S$.

Proposition 1

Let $S \neq N$ be non-empty. If (a), (b), or (c) holds, then (1) holds, and thus, there exists a stable set K^S for $(\{0\} \cup N, v, P^S)$ such that $x_0 = s(W(s) - L(s)) - n(L(0) - L(s))$ for any $x \in K^S$.

(a) $s \leq n/2$.

(b) $W(1) > sW(s)$ for all $s = 2, ..., n$ and $L(s) = 0$ for all $s = 1, ..., n-1$. (drastic innovation)

(c) A linear Cournot industry

4. Analysis: Lower bound

Proposition 2

Let $S \subset N$ be non-empty. If $H^S(z)$ is externally stable, then

$$sW(s) + (n-s)L(s) - nL(0) \leq z.$$

- Fix an arbitrary $\bar{y} \in I^S$ with

$$\bar{y}_0 < sW(s) - (n-s)L(s) - nL(0),$$

  and take $y' \in I^S$ s.t. $\bar{y}_0 < y'_0 < sW(s) + (n-s)L(s) - nL(0)$ and $y'_i = W(s) - y'_0/s$ for each $i \in S$.
- Suppose that there existed some $y \in H^S(\bar{y}_0)$ s.t. $y \succ y'$.
- Since $\bar{y}_0 < y'_0$, it is not true that $y \succ_{\{0\}\cup T} y'$ for any $T \subset N$ and any $y \in H^S(\bar{y}_0)$.

- Then, there exists some nonempty $T \subset S$ s.t. $v(T \cup N \setminus S) = (t+n-s)L(n-(t+n-s)) > \sum_{i \in T} y'_i + (n-s)L(s)$.

  By Assumptions, it is shown that for all $t = 1, ..., s$,
  $\max\{(t+r)L(n-(t+r)) - rL(s) |\ 0 \leq r \leq n-s\}$
  $= (t+n-s)L(s-t) - (n-s)L(s)$.

- That is,

$$
\begin{aligned}
(t+n-s)L(s-t) - (n-s)L(s) &> \sum_{i \in T} y'_i \\
&= tW(s) - \frac{t}{s} y'_0 \\
&> tW(s) - \frac{t}{s}\left(sW(s) - nL(0) + (n-s)L(s)\right) \\
&= \frac{nt}{s} L(0) - \frac{t(n-s)}{s} L(s).
\end{aligned}
$$

However,

$$\begin{aligned}
&\left(\frac{nt}{s}L(0) - \frac{t(n-s)}{s}L(s)\right) - ((t+n-s)L(s-t) - (n-s)L(s)) \\
&= \frac{nt}{s}L(0) - (t+n-s)L(s-t) + (n-s)\left(1 - \frac{t}{s}\right)L(s) \\
&> \left(\frac{nt}{s} - t + n - s\right)L(0) + (n-s)\left(1 - \frac{t}{s}\right)L(s) \\
&= \frac{1}{s}(n-s)(s+t)L(0) + (n-s)\left(1 - \frac{t}{s}\right)L(s), \\
&> 0,
\end{aligned}$$

contradiction. Therefore, $y \succ y'$ is not true for any $y \in H^S(\bar{y}_0)$.
Q.E.D.

- Assumption (iii): $W(k) - W(k+1)$ is non-increasing in $k = 1, ..., n-1$

Proposition 3

Let $S \subset N$ be non-empty. If $H^S(x_0)$ is externally stable, then

$$x_0 < \max_{0 \le t \le n} \max_{0 \le r \le \min\{t, n-s\}} \left(tW(t) - (t-r)W(s) - rL(s) \right),$$

under Assumptions (I), (II), and (III)

Proposition 4

Let $S \subset N$ be non-empty. If

- $n/2 < s \leq (2n+1)/3$ and
- $sW(s) + (n-s)L(s) - nL(0) > \max_{t=1,\ldots,n-s} t(W(t) - L(s))$,

then there exists no stable set $K^S \subset H^S(z)$ for any z, under Assumptions (I), (II) and (III).

Example

$n = 3$, $s = 2$.

$W(1)$	$W(2)$	$W(3)$	$L(0)$	$L(1)$	$L(2)$
30	20	10	2	1	0

Theorem 3

Let $x_0^* = n(W(n) - L(0))$. If there exists some $t = 1, ..., n-1$ such that

$$tW(t) - n(W(n) - L(0)) > tW(n); \tag{2}$$

$$nL(0) - (n-k)L(k) > \frac{k(nW(n) - (t-k)L(n-t+k))}{n-t+k}$$
$$\text{for all } k = 1, ..., t, \tag{3}$$

then $\{x_0^*\} \times C(v_{x_0^*}^N)$ is a stable set in $(\{0\} \cup N, v)$.

Let $c^* = (n(W(n) - L(0)), L(0), \ldots, L(0))$. For each $i \in \{0\} \cup N$, define

$$L^i = \{x \in I^N | x_i \in [L(\rho(1)), c_i^*], x_j = \ell_j^i(x_i)\ (\forall j \in (\{0\} \cup N) \setminus \{i\}),$$
$$\textstyle\sum_{j \in (\{0\} \cup N) \setminus \{i\}} \ell_j^i(x_i) = nW(n) - x_i\},$$

where, for each $j \in (\{0\} \cup N) \setminus \{i\}$, $\ell_j^i(x_i)$ satisfies

- $\ell_j^i(x_i)$ is continuous and decreasing in x_i,
- $\ell_j^i(c_i^*) = c_j^*$.

Theorem 4

If $\{1, n\} = \arg\max_{s=1,\ldots,n} s(W(s) - L(0))$ and $2L(n-2) \geq L(0)$, then $K = \bigcup_{i \in \{0\} \cup N} L^i$ is a stable set. ($C^N = \bigcap_{i \in \{0\} \cup N} L^i$.)

5. Symmetric payoffs

- The nonexistence problem impedes the stable set from applications.
- To avoid this, we restrict imputations on symmetric payoffs for licensees.

$$\tilde{I}^S = \{x \in I^S \mid x_i = x_j \text{ for all } i, j \in S\}.$$

  - By $\tilde{I}^S \subset I^S$, we can employ the same dominance relation on $\tilde{I}^S$.
- Two justifications.
  - Equity among licensees. (ref. Rabin, 1993, AER)
  - Bargaining on the uniform fee (price) for the license.

$\Rightarrow$ The symmetric stable sets always exist, and they are completely characterized.

5. Symmetric payoffs: Definition

Stable set

We say $\tilde{K}^S \subset \tilde{I}^S$ is a stable set iff $\tilde{K}^S$ satisfies the followings.

Internal stability For any $x, x' \in \tilde{K}^S$, $x \succ x'$ does not hold.

External stability For any $x' \in \tilde{I}^S \setminus \tilde{K}^S$, there exists some $x \in \tilde{K}^S$ such that $x \succ x'$.

Core

We say $\tilde{C}^S \subset \tilde{I}^S$ is the core iff

$$\tilde{C}^S = \{x \in \tilde{I}^S | \text{ there exists no } x' \text{ such that } x' \succ x\}.$$

5. Symmetric payoffs: Result

$$sW(s) + (n-s)L(s) - nL(0) < \max_{t=1,\dots,n-s} t(W(t) - L(s)). \qquad (*)$$

(Similar to the existence condition in the former model.)

Theorem 5

- If (*) is satisfied, then $\tilde{K}^S$ is a stable set iff $\tilde{K}^S = \{x\}$ s.t.

$$sW(s) + (n-s)L(s) - nL(0) \leq x_0 \leq \max_{t=1,\dots,n-s} t(W(t) - L(s)).$$

- If (*) is **NOT** satisfied, then

$$\left\{ x \in \tilde{I}^S \;\middle|\; \begin{array}{l} \max_{t=1,\dots,n-s} t(W(t) - L(s)) \\ \quad \leq x_0 \leq sW(s) + (n-s)L(s) - nL(0) \end{array} \right\}$$

is the sym core and it is the unique sym stable set.

5. The acceptance of payments.

Finally check whether each licensee obtains $L(s-1)$ or more.
Fix $x_0 = sW(s) - (n-s)L(s) - nL(0)$ $(< s(W(s) - L(0)))$.

$\Rightarrow$ There are fees all licensees accept,
when $x_0 = sW(s) + (n-s)L(s) - nL(0)$.

- For $x \in \tilde{K}^S$, each $i \in S$ obtains more than $L(0)$.

6. Earlier results: Bargaining set family

Solutions in the bargaining set require stability concepts.

(1) nucleolus $\in$ nonempty core $\subset$ bargaining set

(2) nucleolus $\in$ kernel $\subset$ bargaining set

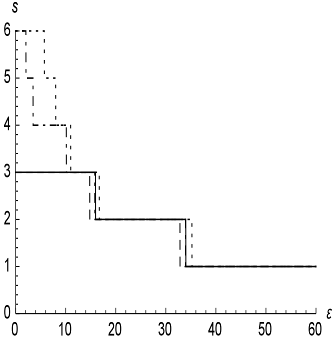
- Watanabe and Muto (2008, IJGT); Bargaining sets are singletons under some P^S. (Almost all cores are empty.)
- Kishimoto, Watanabe, and Muto (2011, MSS); As the number of firms tends to infinity, the bargaining set converges to a point.
  - It exactly coincides with the non-cooperative outcomes shown in Kamien, Oren, and Tauman (1992, JME).
- Kishimoto and Watanabe (2015, under review); The kernel is always a singleton under each P^S.

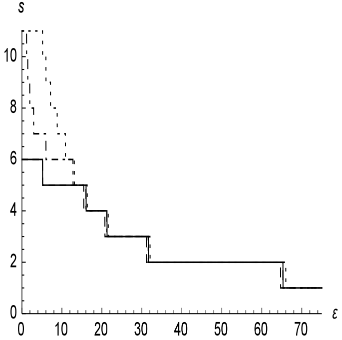
6. Numerical analysis: A linear Cournot market

- There are n firms that produce a homogeneous product.
- The inverse demand function of the market is $p(q) = \max\{0, a - q\}$, where $c < a < \infty$ and q denotes the total production level in the market.
- The patent holder has a patent of a new technology that reduces the unit cost of production from c to $c - \varepsilon$, where $0 < \varepsilon < c$.
- The Cournot equilibrium gross profits $W(t)$ and $L(t)$ of each licensee and each non-licensee at stage (iii) are given as follows:

$$W(t) = \begin{cases} [(a - c + (n - t + 1)\varepsilon)/(n+1)]^2 & \text{if } t \leq (a-c)/\varepsilon \\ [(a - c + \varepsilon)/(t+1)]^2 & \text{if } t \geq (a-c)/\varepsilon, \end{cases}$$

$$L(t) = \begin{cases} [(a - c - t\varepsilon)/(n+1)]^2 & \text{if } t \leq (a-c)/\varepsilon \\ 0 & \text{if } t \geq (a-c)/\varepsilon. \end{cases}$$





Solid line stands for $\hat{s}$; Dashed line represents k^*; Dot-dashed line denotes $\bar{k}$. The parameters are set as follows; $n = 6$, $a = 100$, and $c = 60$ for the upper two diagrams, while $n = 11$, $a = 150$, and $c = 75$ for the lower two diagrams.

6. Values of innovation

- Let $\bar{k}$ denote the number (integer) of licensees which maximizes the producer surplus (the value of innovation in the industry)

$$sW(s) + (n - s)L(s) - nL(0).$$

- Let k^* be the number of licensees which maximizes the patent holder's revenue (private value of innovation) by means of take-it-or-leave-it offer.

- Let $\hat{s}$ be the number of licensees which maximizes the patent holder's revenue when we apply the kernel as the solution to negotiations in stage (ii).

Result 1: Numerical Analysis: Kishimoto and Watanabe (2015)

When a cost-reducing technology is licensed to firms operating in a linear Cournot market, our numerical analysis indicates that $\hat{s} < \bar{k} < k^*$.

The innovation is not utilized efficiently via licensing by means of both kernel and take-it-or-leave-it offer.

6. Implication: Continued

- We can apply another solution concept to stage (ii), von Neumann-Morgenstern stable set (vN-M stable set).

Result 2: General Model with Some Restrictions: This Paper

When the patent holder accepts a payoff in vN-M stable set for each coalition structure P^S, the number of licensees which mazimizes his or her renenue via negotiations coincides with $\bar{k}$.

The innovation may be utilized efficiently via licensing by means of vNM stable sets.

Thanks.