A Numerical Study with Experimental Data on Risk-Averse Subcontractors in Procurement Auctions with Subcontract Bids

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- My student and I were preparing for a presentation on the incentive schemes for information exchange among employees of a leading company in Japan, which may be called internal information exchange market.
- Unfortunately, it is still incomplete.
- This time, I will propose a methodological issue for the experiment which is supposed to be conducted in the near future.

1. Introduction: Subcontract Auctions

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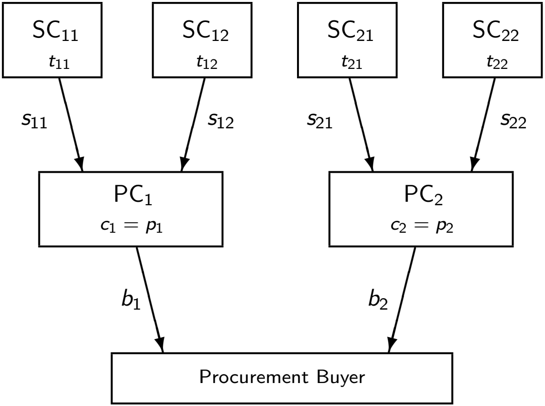
- In a subcontract ("reverse") auction, there are
  - a PC as auctioneer
  - SCs as bidders

A Simple Model



- SC_{ij}, $j = 1, 2$,
  - cost: t_{ij}, *iid*,
  - bid: s_{ij} to PC_i
  - payment:
    $p_{ij}^{1st} = \min\{s_{i1}, s_{i2}\}$
    $p_{ij}^{2nd} = \max\{s_{i1}, s_{i2}\}$
    if s_{ij} is lowest;
    $p_{ij} = 0$ otherwise.
- 2 PCs, $i = 1, 2$
  - payment to SC:
    $p_i = p_{ij}^{1st}$ or p_{ij}^{2nd}
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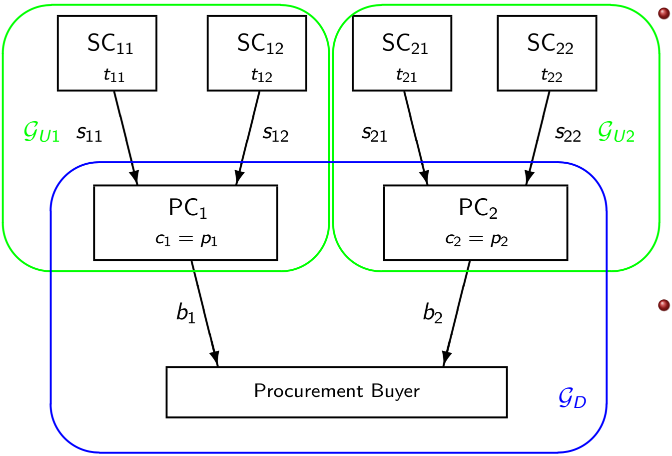

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    1. (i) SCs submit higher bids in FPA (1st price reverse auction) than in SPA (2nd price reverse auction), regardless of their degree of risk-aversion. (ii) In FPA, SCs lower their bids as their degree of risk-aversion increases.

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- Experimental Results

  All predictions were statistically observed except Prediction 2.

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- Suggestions
  In experiments for data exchange or trades of information which will be conducted, construct first a subject pool in which the coefficients of subjects' risk attitudes are measured in advance.
  - It is difficult to assume that subjects' risk attitude are kept intact during a session.
  - Controlling subjects' (moderate or extreme) risk attitudes $\Rightarrow$ Robustness check of the performance of (internal or formal) Data Exchange Markets.

2. Theoretical Study: Subcontractors' Bidding Function

$s_{ik} = s(t_{ik})$; bidding function of subcontractors.

- t_{ik} is independently and identically distributed over $[\underline{t}, \bar{t}]$.
- Subcontractors are risk-averse. Subcontractor SC_{ik} obtains utility $u(y) = y^{r_{ik}}$, when it receives income y; $(1 - r_{ik}) \in (0, 1)$ is the risk aversion coefficient (constant relative risk aversion, CRRA). For any SC_{ik}, let $r_{ik} = r$ for simplicity (for the theoretical part).
- A dominant strategy (equilibrium bidding function of subcontractors) in SPA for each SC_{ik} is

$$s^{**}(t_{ik}) = t_{ik}, \tag{1}$$

regardless of any risk aversion coefficients

- In FPA, assuming that PCs use an identical bidding function, each SC_{ik} determines his or her bid s_{ik} so as to maximize his or her expected utility

$$
\begin{aligned}
&(s_{ik}-t_{ik})^r \mathrm{Prob}(s_{ik} < \min(s_{ik'}, s_{j1}, s_{j2})) \\
&= (s_{ik}-t_{ik})^r \mathrm{Prob}(s^{-1}(s_{ik}) < t_{ik'})^3 \\
&= (s_{ik}-t_{ik})^r [1-\mathrm{Prob}(s^{-1}(s_{ik}) \geq t_{ik'})]^3 \\
&= (s_{ik}-t_{ik})^r [1-\frac{s^{-1}(s_{ik})-\underline{t}}{\bar{t}-\underline{t}}]^3,
\end{aligned}
$$

- The first order condition (hereafter, FOC) is $s'(t_{ik})r(s_{ik}-t_{ik})^{r-1}[\bar{t}-t_{ik}]^3 = 3(s_{ik}-t_{ik})^r[\bar{t}-t_{ik'}]^2$.
- The symmetric equilibrium bidding function of subcontractors in FPA is

$$
s^*(t_{ik}) = t_{ik} + \frac{\bar{t}-t_{ik}}{3+r}r. \tag{2}
$$

Prime Contractors' Bidding Function

$b_i = b(c_i)$; bidding function of prime contractors.

- Prime contractors are risk-neutral.
- If SPA is used in subcontract auctions, the cost c_i of PC_i is independently and identically distributed over $[\underline{c}, \bar{c}] = [\underline{t}, \bar{t}]$, because $s_{i,k}^{**}(t_{i,k}) = t_{i,k}$.
- Each PC_i determines its bid b_i so as to maximize the expected payoff

$$
\begin{aligned}
&(b_i - c_i)\mathrm{Prob}(b_i < b_j) = (b_i - c_i)\mathrm{Prob}(b^{-1}(b_i) < c_j) \\
&= (b_i - c_i)\mathrm{Prob}(b^{-1}(b_i) < \max(t_{j1}, t_{j2})) \\
&= (b_i - c_i)[1 - \mathrm{Prob}(b^{-1}(b_i) \geq \max(t_{j1}, t_{j2}))] \\
&= (b_i - c_i)[1 - \mathrm{Prob}(b^{-1}(b_i) \geq t_{j1})\mathrm{Prob}(b^{-1}(b_i) \geq t_{j2})] \\
&= (b_i - c_i)[1 - (\frac{b^{-1}(b_i) - \underline{c}}{\bar{c} - \underline{c}})^2].
\end{aligned}
$$

- The FOC is
$b'(c_i)[(\bar{c} - \underline{c})^2 - (c_i - \underline{c})^2] - 2b_i(c_i - \underline{c}) = -2c_i(c_i - \underline{c})$.
- The symmetric equilibrium bidding function of the prime contractors in SPA is

$$b^{**}(c_i) = \frac{(2/3)(\bar{c}^3 - c_i^3) - \underline{c}(\bar{c}^2 - c_i^2)}{(\bar{c} - \underline{c})^2 - (c_i - \underline{c})^2}, \tag{3}$$

where $c_i = \max\{s_{i1}, s_{i2}\}$ is NOT a function of r. (See SCs' bidding function (1).)

- When FPAs are used as subcontract auctions, the cost $c_j = p_j = \min(s^*(t_{j1}), s^*(t_{j2}))$ of PC_i is independently and identically distributed over $[\underline{c}, \bar{c}]$, where $\underline{c} = (3\underline{t} + \bar{t}r)/(3 + r)$ and $\bar{c} = \bar{t}$, given the SCs' bidding function (2).
- Each PC_i determines its bid b_i so as to maximize the expected payoff

$$\begin{aligned}
&(b_i - c_i)\mathrm{Prob}(b_i < b_j) = (b_i - c_i)\mathrm{Prob}(b^{-1}(b_i) < c_j) \\
&= (b_i - c_i)\mathrm{Prob}(b^{-1}(b_i) < \min(s^*(t_{j1}), s^*(t_{j2}))) \\
&= (b_i - c_i)\mathrm{Prob}(b^{-1}(b_i) < s^*(t_{j1})) \\
&\qquad \times\mathrm{Prob}(b^{-1}(b_i) < s^*(t_{j2})) \\
&= (b_i - c_i)\mathrm{Prob}(b^{-1}(b_i) < \frac{3t_{i1} + \bar{t}r}{3 + r}) \\
&\qquad \times\mathrm{Prob}(b^{-1}(b_i) < \frac{3t_{i2} + \bar{t}r}{3 + r}) \\
&= (b_i - c_i)[1 - \frac{((3 + r)b^{-1}(b_i) - \bar{c}r)/3 - \underline{c}}{\bar{c} - \underline{c}})]^2.
\end{aligned}$$

- Let $\beta(c_i) = ((3+r)c_i - \bar{c}r)/3$ $(< \bar{c})$.
- The FOC is
$3b'(c_i)(\bar{c} - \beta(c_i))^2 - 2(b_i - c)(\bar{c} - \beta(c_i))(3+r) = 0$, i.e.,
$b'(c_i)(\bar{c} - c_i) - 2(b_i - c) = 0$.
- The symmetric equilibrium bidding function of prime contractors in FPA is

$$b^*(c_i) = \frac{\bar{c}^3 - c_i^2(3\bar{c} - 2c_i)}{3(\bar{c} - c_i)^2}, \tag{4}$$

where $c_i = \min\{s_{i1}, s_{i2}\}$ is a function of r. (See SCs' bidding function (2).)

Theoretical Predictions

Suppose that subcontractors are equally risk-averse.

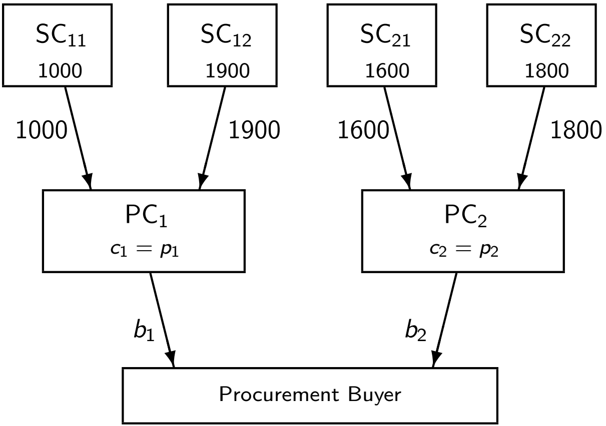
Prediction 1. (i) Subcontractors submit higher bids in FPA than in SPA, regardless of their degree of risk-aversion. (ii) In FPA, subcontractors lower their bids as their risk-aversion increases.

Prediction 2. SPA renders higher expected profits to prime contractors than FPA.

- Let t^* be the cost of the subcontractor who is awarded the subcontract work.
- We say that an allocation of subcontract work is ex post efficient if the social surplus $V - t^*$ is maximized.
    - In ex post efficient allocations, subcontractor SC_{ik} is awarded the subcontract work if and only if $s_{ik} < \min(s_{ik'}, s_{j1}, s_{j2})$, where $k' \neq k$ and $j \neq i$.

Prediction 3. If the subcontractors are equally risk-averse, then the ex post efficient allocation of subcontract work is achieved when the subcontract auctions are FPAs, whereas it is not necessarily achieved when the subcontract auctions are SPAs.

Allocative efficiency in SPA: An example



DWL generated by SPA is 600 (risk-neutral PCs and SCs).

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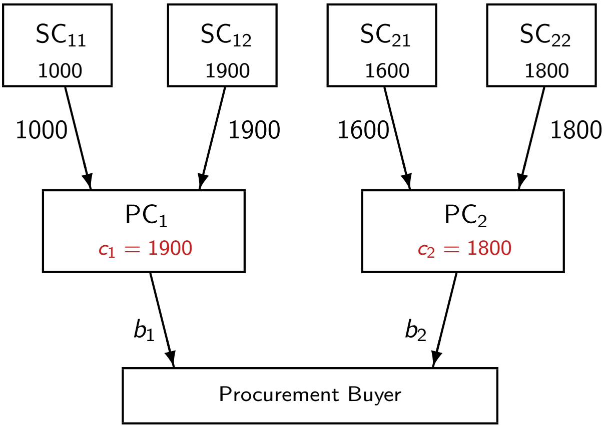

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3. Numerical Study: The Experiment

- In total, 16 experimental sessions were conducted from January 2010 to March 2014.
- Subjects were recruited from all over the campuses at University of Tsukuba and Osaka University.
- SC's cost $\sim U[1,000, 2,000]$, procurement buyer's reserve=2,000.

	no. of PCs	PCs played by machine	no. SCs for each PC	no. subcont. auctions	no. of periods	no. subjects per period
Subsession 1	1	—	2	20	10	4
Subsession 2	2	Yes	2	20	10	4
Subsession 3	2	No	2	40	20	6

- Subjects were randomly matched and assigned to SCs or PCs at the beginning of every period.
- Machine bidders were assumed to be risk-neutral.

Subcontractors' Bids

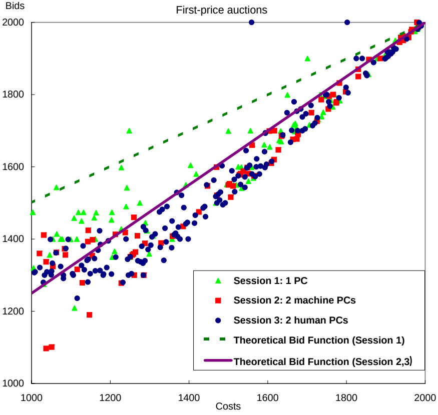

Figure: Typical Bids: Risk Aversion (Left)

Experimental Results

- PC's profit: Prediction 2 was not observed at the 5% level of significance; $p = 0.1155$ for the permutation test in subsession 2 where machine bidders played as prime contractors.
  - FPA: the average was 215.4 with standard deviation 23.4.
  - SPA: the average was 251.6 with standard deviation 142.5.
- DWL: Prediction 3 was statistically confirmed in subsession 2; $p = 0.040$ for the permutation test.

- Given s_{ik}, t_{ik}, and $\bar{t} = 2000$, the value of r_{ik} can be computed from the experimental data using formula (2); the average value is the estimate of the common value of r.
- The average value of r_{ik} was 0.4596 with standard deviation 0.4124 in a session, while it was 0.5870 with standard deviation 0.4653 in another session. The average is about 0.5.
- If subjects were equally risk-averse, then Prediction 2 would be surely observed at the 1% level of significance, because the p-values for $r = 0.5$. (SCs' costs were taken from the values realized in the experiment.)

Table: Prime contractors' profits simulated in the FPA with the value of CRRA coefficients and those simulated in the SPA.

	$r = 1.0$	$r = 0.9$	$r = 0.8$	$r = 0.7$	$r = 0.6$
mean	200.0062	205.1346	210.5329	216.223	222.2292
std dev	38.39252	39.37695	40.41318	41.50543	42.65836
p-value	$<$0.00001	$<$0.00001	0.0000129	0.0000712	0.000377

	$r = 0.5$	SPA
mean	228.5786	271.2249
std dev	43.87717	141.0522
p-value	0.001889	–

[a] The p-values for the permutation test: FPA vs. SPA

Numerical Inference

- Assumption A: Examine two extreme patterns in distributions of CRRA coefficients (the average is 0.5):
  (a) $r_{11} = 0.4$, $r_{12} = 0.4$, $r_{21} = 0.8$, and $r_{22} = 0.8$.
  (b) $r_{11} = 0.8$, $r_{12} = 0.4$, $r_{21} = 0.8$, and $r_{22} = 0.4$.
- Assumption B: Each subcontractor believes that the other subcontractors were as risk-averse as he or she is.
- Given the data of subcontractors' costs, the profits of prime contractors can be generated using the bidding functions (1), (2), (3), and (4).

- case (a): $p = 0.1219$ for the permutation test: No significant difference in PC's profit between the FPA and SPA.
  - the average profit of the winning prime contractors was 218.0 with standard deviation 37.5 when the subcontract auctions were FPAs, whereas it was 255.6 with standard deviation 122.1 when the subcontract auctions were SPAs.
- case (b): similar results.
- This numerical computation thus replicated the experimental results, which were
  - FPA: the average was 215.4 with standard deviation 23.4.
  - SPA: the average was 251.6 with standard deviation 142.5.

Conclusion. It is plausible that unclear results observed in the previous experimental sessions was due to the presence of extreme patterns in the distributions of risk aversion coefficients among the subjects.

4. Conclusion: Summary

- For verifying the theoretical predictions in the experiment, the difficulty did not lie in the bidding behavior of risk-averse subjects, but it was controlling the distribution of subjects' risk attitudes.

Remarks: Discussion

- Truly, it is difficult to assume that subjects' risk attitude are kept intact during a session. $\Rightarrow$ Cox et al. (1985) proposed the ex post adjustments of subjects' risk attitudes.

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- It is, however, never difficult to construct a subject pool in which subjects' risk aversion coefficients are measured in advance.
  - moderate risk aversion coefficients
    $\Rightarrow$ verification of the validity of theoretical predictions in (internal or formal) Data Exchange Markets
  - extreme risk aversion coefficients
    $\Rightarrow$ robustness check of the performance of those markets

Thank you for listening.

In the next opportunity, I will report the experimental results of "internal information exchange market" in a Japanese company.