Asymptotic Results of vNM Stable Sets of a Patent Licensing Game: Revenue Maximization and Fair Distribution in a General Cournot Market

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1. Introduction
Cooperative Approach

1. What Had Been Done: for cooperative interpretations of non-cooperative outcomes

process (cost-reducing) innovation in a general Cournot market (Kamien-Oren-Tauman (KOT), 1992, JME) $\cdots$ asymptotic results in a non-cooperative model

a general cooperative model with coalition structures (Watanabe-Muto (WM), 2008, IJGT)

Previous Results

- (Davis-Maschler) bargaining set and core (Watanabe-Muto 2008, IJGT)
  - The core is empty for every coalition structure in any Cournot markets.
- the bargaining set asymptotically reaches the same outcomes as those in KOT 1992. (Kishimoto-Watanabe-Muto, 2011, MSS)
- kernel = nucleolus (Kishimoto-Watanabe, 2017, MSS)
  - The existence conditions for the stable sets (Hirai-Watanabe, 2018, MSS)

1. Main Results of This Paper

Present Results: asymptotic results of (vNM) stable sets

(1) Some type of stable sets asymptotically reaches the same outcomes as those in KOT 1992.

- revenue maximization

(2) Another type does not, but in the limit (when the # of firms is sufficiently large) the Aumann-Drèze-Shapley (ADS) value of the patent holder can coincide with the revenue he receives as his payoff in the stable sets.

- fair distribution

··· **without** a coalition formation stage (Tauman-Watanabe, 2007, ET; the **grand coalition** is formed, and in the Shapley value the patent holder can take all in a **linear** Cournot market.)

1. What is to be done: for richer future analyses

A new model: farsighted stable sets in an abstract game (Hirai-Watanabe-Muto, 2019, GEB)

presentation slides:
http://labs.kbs.keio.ac.jp/naoki50lab/HitU_patent_FSS.pdf

- Players' preferences can be defined over outcomes, not only on their own payoffs. No need for defining any characteristic functions.
  $\Rightarrow$ Other-regarding or social preferences and fairness notions are tractable more directly.
  - In the paper, authors did not define those things but simply used the individual payoff for each player.
- A remaining question for future research:
  What occurs in a mixture of myopic and farsighted players?

If time permits (probably no), this part may be referred to in this talk.

2. The Model
Watanabe-Muto 2008

2. Patent licensing game: stage (i)

Process innovation and product innovation can be treated in this general model.

- $N_n = \{1, \ldots, n\}$: the set of symmetric firms ($2 \leq n < \infty$)
- player 0: external patent holder ($\{0\} \cup N_n$: the set of players)
- 3-stage game

stage (i): The patent holder selects a set $S_n (\subset N_n)$ of firms for license negotiations.

- Coalition $\{0\} \cup S_n$ forms only for license negotiation.
- $P^{S_n} = \{\{0\} \cup S_n, \{\{i\}\}_{i \in N_n \setminus S_n}\}$: permissible coalition structure

2. Patent licensing game: stage (ii)

stage (ii): Firms in S_n negotiate license fees with the patent holder and make payments (by means of fixed fee).

- Check the acceptance of payments by each firm after finding the bargaining outcome.
- Analyze the negotiation for each coalition structure P^{S_n}, assuming that all firms in S_n are given a license for simplicity.

2. Patent licensing game: stage (iii)

stage (iii): Knowing that which firms are licensed, each firm in N_n competes in the market. (Any cartels are prohibited.)

- When t_n firms are licensed, each licensee obtains the gross profit $W(t_n)$ and each non-licensee who uses an old technology obtains the gross profit $L(t_n)$.
- Assume that $W(t_n) > L(0) > L(t_n)\ \forall t_n = 1, \ldots, n-1, (n)$. Negative eternality arises in $L(t_n)$
- Each firm accepts the payment if it is $L(t_n - 1)$ or more.

2. A general Cournot market in stage (iii)

Kamien-Oren-Tauman (1992)

- Each firm i produces q_i unit of a homogeneous commodity with the unit cost of production c. Let $q = \sum_{i \in N_n} q_i$.
- The inverse demand function of the market is $p = P(q)$, where $P(0) > c$. The demand function is denoted by $Q(p)$
  - $P(q)q$ is strictly concave in q.
  - $Q(p)$ is decreasing, differentiable. The price elasticity $\eta(p) = -pQ'/Q$ is non-decreasing in p.
- The patent holder has a patent of a new technology that reduces the unit cost of production from c to $c - \varepsilon$, where $0 < \varepsilon < c$.
- Assume $K = \dfrac{c}{\epsilon\eta(c)} > 1$: non-drastic innovation.

2. A general Cournot market in stage (iii), continued

- The Cournot equilibrium gross profits $W(t_n)$ and $L(t_n)$ of each licensee and each non-licensee at stage (iii) are given as

$$W(t_n) = \begin{cases} -\dfrac{(p-c+\varepsilon)^2}{P'} & \text{if } 1 \leq t_n \leq K \\ \dfrac{(p-c+\varepsilon)Q(p)}{t_n} & \text{if } K \leq t_n \leq n, \end{cases}$$

$$L(t_n) = \begin{cases} -\dfrac{(p-c)^2}{P'} & \text{if } 0 \leq t_n \leq K \\ 0 & \text{if } K \leq t_n \leq n. \end{cases}$$

Note that for $0 < t_n \leq K$, $W(1_n) > \cdots > W(t_n) > \cdots > W(n) > L(0_n) > \cdots L(t_n) \cdots > L(K) = \cdots = L(n-1) = 0$.

2. A bargaining game in stage (ii)

$(\{0\} \cup N_n, v, P^{S_n})$: a game with a coalition structure

$\cdots$ Aumann and Drèze (1974, IJGT)

- $v : 2^{\{0\} \cup N} \to \mathbb{R}$; a characteristic function
  - $v(\{0\}) = v(\emptyset) = 0$.
  - $v(\{0\} \cup T_n) = t_n W(t_n)$ for all nonempty $T_n \subset N_n$.
  - $v(T_n) = t_n L(n - t_n)$ for all nonempty $T_n \subset N_n$.

I^{S_n}: the set of imputations under P^{S_n}, where

$$I^{S_n} = \left\{ \begin{array}{c} x^n = (x_0^n, x_1^n, \ldots, x_n^n) \\ \in \mathbb{R}^{n+1} \end{array} \;\middle|\; \begin{array}{l} x_0^n + \sum_{i \in S} x_i^n = s_n W(s_n), \\ x_0^n \geq v(\{0\}) = 0, \\ x_i^n \geq v(\{i\}) = L(n-1) \; \forall i \in S_n, \\ x_i^n = L(s_n) \; \forall i \in N_n \setminus S_n \end{array} \right\}$$

Kishimoto-Watanabe-Muto (2011): A sequence of $t_n = |T_n|$ is said to converge to an integer t, if there exists n' such that for all $n > n'$ we have $|T_n| = t$, which is written as

$$t = \lim_{n\to\infty} t_n.$$

Lemma A

(a) If $t \leq K$, then $\lim_{n\to\infty} t_n W(t_n) = t\varepsilon Q(c)/K$.
(skip the case for $K < t_n < \infty$)

(b) If t_n diverges, then $\lim_{n\to\infty} t_n W(t_n) = 0$.

(c) For any t_n, $\lim_{n\to\infty} t_n L(n - t_n) = 0$, regardless of whether t_n converges or diverges.

Lemma B

Let $s_n^{'}$ be such that $s_n^{'} W(s_n^{'}) \geq s_n W(s_n)$ for $s_n = 1, \ldots, n$. Then, $\lim_{n\to\infty} s_n^{'} = K$.

The bargaining set for P^{S_n} is denoted by M^{S_n}. (See the paper for the definition.)

Note 1: Kishmoto-Watanabe-Muto (2011)

Suppose that $S_n \subsetneq N_n$. Take any $x^n \in M^{S_n}$. Then, in the general Cournot market, $\lim_{n\to\infty} x_0^n = \lim_{n\to\infty} s_n W(s_n)$ and $\lim_{n\to\infty} x_i^n = 0$ for all $i \neq 0$.

This result completely coincides with the one shown in Kamien-Oren-Tauman (1992).
($\cdots$ In Tauman-Watanabe (2007), the grand coalition is formed and $n - K$ licensees stop their production.)

3. The Stable Sets
Hirai-Watanabe 2018

3. Dominance relation

Dominance relation

Let $x^n, y^n \in I^{S_n}$.

We say that x^n dominates y^n via $T_n \subset \{0\} \cup N_n$, denoted by $x^n \succ_T y^n$, iff

- $T_n \cap (\{0\} \cup S_n) \neq \emptyset$,
- $\sum_{i \in T_n} x_i^n \leq v(T_n)$,
- $x_i^n > x_i^n \ \forall\ i \in T_n \cap (\{0\} \cup S_n)$.

We say that x^n dominates y^n, denoted by $x^n \succ y^n$, iff x^n dominates y^n via some $T_n \subset \{0\} \cup N_n$.

3. Stable sets

Stable set

$K^{S_n} \subset I^{S_n}$ is a stable set for a bargaining game $(\{0\} \cup N_n, v, P^{S_n})$ if K^{S_n} satisfies the following conditions.

Internal stability: For any $x^n, y^n \in K^{S_n}$, $x^n \succ y^n$ does not hold.

External stability: For any $x^n \in I^{S_n} \setminus K^{S_n}$, there exists some $x^n \in K^{S_n}$ such that $x^n \succ y^n$.

- For any x_0^n $(0 \leq x_0^n \leq s_n W(s_n))$, define $H^{S_n}(x_0^n) = \{z^n \in I^{S_n} | \ z_0^n = x_0^n\}$.
- Since we are interested in the PH's revenue, we concentrate on a stable set K^S such that $K^{S_n} \subset H^{S_n}(x_0^n)$ for some x_0^n.

- The core C^{S_n} for a bargaining game $(\{0\} \cup N_n, v, P^{S_n})$ is defined as

$$C^{S_n} = \{x^n \in I^{S_n} | \nexists y^n \in I^{S_n}, y^n \succ x^n\}.$$

Note 2: Watanabe-Muto 2008

(1) For any non-empty $S_n \subset N_n$, if $S_n \neq N_n$, then $C^{S_n} = \emptyset$. $C_n^N \neq \emptyset$ if and only if $n \in \arg\max_{s_n=1,\ldots,n} s_n(W(s_n) - L(0))$.

(2) In a general Cournot market, $C^{S_n} = \emptyset$ for any permissible coalition structure P^{S_n}.

3. A key step: reduced game

Given $(\{0\} \cup N_n, v_n, P^{S_n})$ and $x_0^n \in [0, s_n W(s_n)]$, let $(S_n, v_{x_0}^{S_n})$ be a reduced game s.t.

$$v_{x_0^n}^{S_n}(T_n) = \begin{cases} 0 & \text{if } T_n = \emptyset \\ s_n W(s_n) - x_0 & \text{if } T_n = S_n \\ (t_n + n - s_n)L(s - t_n) - (n - s_n)L(s_n) & \text{if } T_n \subset S_n \end{cases}$$

- The core $C(v^{S_n}_{x^n_0})$ of the reduced game $(S_n, v^{S_n}_{x^n_0})$ is large if and only if for any non-empty $T_n \subset S_n$, there exists some $z^n \in C(v^{S_n}_{x^n_0})$ such that $\sum_{i\in T_n} z^n_i \leq v^{S_n}_{x^n_0}(T_n)$.

Lemma C (external stability)

Let $S_n \subset N_n$ be non-empty and $x^n \in I^{S_n}$ be such that

$$s_n W(s_n) + (n - s_n)L(s_n) - nL(0) \leq x^n_0. \tag{1}$$

Assume that $C(v^{S_n}_{x^n_0})$ is large. Let

$$K^{S_n} = \{x^n_0\} \times C(v^{S_n}_{x^n_0}) \times \{(L(s_n), \ldots, L(s_n))\}.$$

Then, for any $z^n \in I^{S_n} \setminus K^{S_n}$ such that $x^n_0 \leq z^n_0$, there exists some $y^n \in K^{S_n}$ such that $y^n \succ z^n$.

3. The existence in the case of $S_n \neq N_n$

- $(S_n, v_{x_0^n}^{S_n})$ is convex: for any $S, T \subset S_n$,
  $v_{x_0^n}^{S_n}(S) + v_{x_0^n}^{S_n}(T) \leq v_{x_0^n}^{S_n}(S)(S \cup T) + v_{x_0^n}^{S_n}(T)(S \cap T)$.
- Every convex game has the large core. (Sharkey, 1982)

Theorem 1

Let $S_n \neq N_n$ be non-empty. If

$$s_n W(s_n) + (n - s_n)L(s_n) - nL(0) \leq \bar{s}_n(W(\bar{s}_n) - L(s_n)), \qquad (2)$$

where $\bar{s}_n \in \arg\max_{t_n=0,\ldots,n-s_n} t_n(W(t_n) - L(s_n))$, then there exists a stable set K^{S_n} for $(\{0\} \cup N_n, v, P^{S_n})$ such that $x_0^n = s_n W(s_n) + (n - s_n)L(s_n) - nL(0)$ for any $x^n \in K^{S_n}$.

Skip the existence in the case of $S_n = N_n$ due to Lemmas A and B: at stage (i), the optimal number of licensees should be less than or equal to K. Condition (2) is satisfied in the linear Cournot market.

4. Asymptotic Results

4. Stable sets with equal treatment

Lemma A

(a) If $t \leq K$, then $\lim_{n\to\infty} t_n W(t_n) = t\varepsilon Q(c)/K$.

(c) For any t_n, $\lim_{n\to\infty} t_n L(n - t_n) = 0$.

Lemma B

Let $s_n^{'}$ be such that $s_n^{'} W(s_n^{'}) \geq s_n W(s_n)$ for $s_n = 1, \ldots, n$. Then, $\lim_{n\to\infty} s_n^{'} = K$.

Treat $K = c/\epsilon\eta(c)$ as an integer. Note that $L(K) = 0$.

Proposition 1

Let $s_n = K$. As $n \to \infty$,
$s_n W(s_n) + (n - s_n)L(s_n) - nL(0) \leq \bar{s}_n(W(\bar{s}_n) - L(s_n))$, where $\bar{s}_n \in \arg\max_{t_n=0,\ldots,n-s_n} t_n(W(t_n) - L(s_n))$, is satisfied, and $\lim_{n\to\infty} x_0^n = \lim_{n\to\infty} s_n W(s_n) + (n - s_n)L(s_n) - nL(0) = \varepsilon Q(c)$ for any $x^n \in \lim K^{S_n}$.

4. The Aumann-Drèze-Shapley value

Let $\varphi^{S_n}(\in \mathbb{R}^{n+1})$ denote the Aumann-Drèze-Shapley value of our bargaining game with a coalition structure P^{S_n}.

- The Aumann-Dréze-Shapley value is player i's average marginal contribution to coalitions in the coalition to which i belongs under a coalition structure P^{S_n}.
- It is interpreted as representing a fair allocation, but in the limit it is not obtained in a stable set K^{S_n}.

Note 3: Kishimoto-Watanabe-Muto (2011)

In the general Cournot market,

$$\lim_{n\to\infty} \varphi_0^{S_n^*} = \frac{\varepsilon Q(c)}{2}, \ \lim_{n\to\infty} \varphi_i^{S_n^*} = \frac{\varepsilon Q(c)}{2K} \text{ if } i \in S_n^*,$$

$$\text{and } \lim_{n\to\infty} \varphi_j^{S_n^*} = 0 \text{ if } j \in N_n \setminus S_n^*. \ (|S_n^*| = K.)$$

Question: Is the AD value (in the limit or not) contained a stable set which is other than the one suggested in Proposition 1?

4. Another type of stable sets: An Example

Treat $K = c/\epsilon\eta(c)$ as an integer for simplicity, instead of using the Gauss symbol. It suffices to show the case of $s_n = K$ for stage (i) by Lemmas A and B.

Proposition 2

Consider the case of $s_n = K = n - 1$. Suppose that (a) $s_n W(s_n) - 2_n s_n L(n - 2_n) \geq W(1_n)$, (b) $2_n W(2_n) \geq s_n W(s_n)$, and (c) $2_n s_n^{'} L(n - 2_n) \geq (s_n^{'} + 1)L(n - (s_n^{'} + 1))$ for any $s_n^{'}$ with $s_n^{'} \leq s_n$. For any ε with $0 \leq \varepsilon \leq 2L(n - 2_n)$, define

$$J^{\varepsilon} = \left\{ x^n \in I^{S_n} \,\middle|\, x_0^n \geq W(1_n), x_1^n \geq 2_n L(n - 2_n), (x_j^n = \varepsilon)_{j=2,\ldots,K} \right\},$$

where $x_{K+1}^n = \cdots = x_n^n = 0$ for any $x^n \in I^{S_n}$. Then, J^{ε} is a stable set. (This type of stable sets disappears in the limit.)

Note that $W(1_n) \leq x_0^n \leq s_n W(S_n) - 2_n L(n - 2_n) - (K - 1)\varepsilon$.

4. Another type of stable sets, cont.

The interpretation of K^{ε}: (1) $K-1$ Licensees do not know the market size and thus prefer a guaranteed amount of payoff ε. (2) After licensing to $K-1$ licensees, the market size is disclosed to the public, and then negotiations on the payment to the patent holder begin with a licensee.

4. Proof: the external stability

Let $y^n \in I^{S_n} \setminus K^\varepsilon$. Assume $s_n = K = n-1$.

- If $y_0^n < W(1_n)$, then $x^n \succ_{\{0,K+1\}} y^n$, where
  $x^n = (W(1_n), A, \varepsilon, \ldots, \varepsilon, 0, \ldots, 0) \in K^\varepsilon$ and
  $A = s_n W(s_n) - W(1_n) - (K-1)\varepsilon$, because $v(\{0, K+1\}) = W(1_n)$.

- If $y_1^n < 2L(n-2_n)$, then $x^n \succ_{\{1,K+1\}} y^n$, where
  $x^n = (s_n W(s_n) - B - (K-1)\varepsilon, B, \varepsilon, \ldots, \varepsilon, 0, \ldots, 0) \in K^\varepsilon$, where
  $B = 2L(n-2_n)$, because $v(\{1, K+1\}) - x_{K+1}^n = 2L(n-2_n)$.

- If $y_j^n < \varepsilon$ $(j = 2, \ldots, K)$, then $x^n \succ_{\{j,K+1\}} y^n$, where
  $x^n = (W(1_n), A, \varepsilon, \ldots, \varepsilon, 0, \ldots, 0) \in K^\varepsilon$
  by $\varepsilon \le v(\{j, K+1\}) - x_{K+1}^n = 2L(n-2_n)$.

4. Proof: the external stability, cont.

- Next, we consider the case of $y^n \in I^{S_n} \setminus K^\varepsilon$ where $y_0^n \geq W(1_n)$, $y_1^n \geq 2L(n-2_n)$, and $y_j^n \geq \varepsilon$. There should exist at least a licensee j such that $j \in \{j' = 2, \ldots, K | y_j^n > \varepsilon\}$ by $y^n \notin K^\varepsilon$.
  - Define $z^n = (y_0^n + B/2, y_1^n + B/2, \varepsilon, \ldots, \varepsilon, 0, \ldots, 0)$, where $B = \sum_{j \in \{j'=2,\ldots,K | y_j^n > \varepsilon\}} (y_j^n - \varepsilon)$.
  - Note that $z^n \in K^\varepsilon$, because $y_0^n \geq W(1_n)$, $y_1^n \geq 2L(n-2_n)$.
  - Let $T_n = \{1, n\}$. Then,

$$\begin{aligned} \textstyle\sum_{i \in \{0\} \cup T_n} z_i^n &= y_0^n + y_1^n + B + y_n^n \\ &= s_n W(s_n) - (K-1)\varepsilon \\ &< s_n W(s_n) = v(\{0\} \cup S_n). \end{aligned}$$

If $v(\{0\} \cup S_n) \leq v(\{0\} \cup T_n)$, i.e., $KW(K) \leq 2_n W(2_n)$ (Assumption (b)), and $z_i^n > y_i^n$ for $i \in T_n$, then $z^n \succ_{\{0\} \cup T_n} y^n$.

■

4. Proof: the internal stability

Fix an arbitrary ε with $0 \leq \varepsilon \leq 2_n L(n - 2_n)$.
Take arbitrary $x^n, y^n \in K^\varepsilon$.

- It is impossible that $x^n \succ_{T_n} y^n$ for any $T_n = \{0\}, \{i\}$ $(i \in S_n)$, because $v(\{0\}) = v(\{i\}) = 0$.
- It is not true that $x^n \succ_{T_n} y^n$ for any T_n s.t. $j \in T_n$ $(j = 2, \ldots, K)$, because $x^n_K = y^n_K = \varepsilon$.
- It is neither true that $x^n \succ_{\{0\} \cup \{1\}} y^n$, because $x^n_0 + x^n_1 = s_n W(s_n) - (K - 1)\varepsilon$.

4. Proof: the internal stability, cont.

- It is, however, possible that $x^n \succ_{\{0\}\cup T_n} y^n$ for some T_n s.t. $T_n \subseteq \{K+1,\ldots,n\}$ because it is not necessarily true that $W(1_n) \geq t_n W(t_n) = v(\{0\}\cup T_n) - \sum_{k\in T_n} x_k^n$.
  - It is impossible by $y_0^n \geq W(1_n)$ if $s_n = K = n-1$, because $|T_n| = 1$.

When $s_n = K = n-1$, $T_n = \{K+1,\ldots,n\} = \{n\}$. Note that $y_1^n \geq 2_n L(n-2_n)$.

- It is impossible that $x^n \succ_{S_n' \cup\{n\}} y^n$ for any $S_n' \subseteq S_n$, if $\sum_{i\in S_n'} y_i^n \geq 2_n L(n-2_n) + (s_n' - 1)\varepsilon \geq (s_n'+1)L(n-(s_n'+1)) = v(S_n'\cup\{n\}) - x_n^n$ by Assumption (c).
  (Note that $\varepsilon \leq 2_n L(n-2_n)$.)

■

Let $n = 3$ and $s_n = K = n - 1 = 2$. Fix $\varepsilon = x_1^n = 2_n L(n - 2_n)$.
Then, the AD value is contained in

$$J^\varepsilon = \left\{ x^n \in I^{S_n} \,\middle|\, x_0^n \geq W(1_n), x_1^n \geq 2_n L(n - 2_n), (x_j^n = \varepsilon)_{j=2,\dots,K} \right\},$$

where $x_{K+1}^n = \cdots = x_n^n = 0$ for any $x^n \in I^{S_n}$.

$$\begin{aligned} \varphi_0^{S_n^*} &= (W(1_n) - L(n - 1_n) + 2_n(W(2_n) - L(n - 2_n)))/3 \\ &= (W(1_n) + 2_n W(2_n) - 2_n L(1_n))/3. \end{aligned}$$

If $\varphi_0^{S_n^*} = 2_n W(2_n) - 2_n(2_n L(n - 2_n)) = 2_n W(2_n) - 4_n L(1_n)$, then $\varphi^{S_n^*}$ is at the edge of J^ε, by $s_n W(s_n) - 2_n s_n L(n - 2_n) \geq W(1_n)$ (Assumption (a)). Note that Assumptions (b) and (c) are always satisfied when $s_n = K = n - 1 = 2$.

4. The AD value and a Stable Set

In the linear Cournot market, where the inverse demand function is $p = \max(0, a - q)$,

- Assumption (a) is satisfied when $s_n = K = n - 1 = 2$, and
- There exist no parameters (a, c, and ε) with which we can obtain $\varphi_0^{S_n^*} = 2_n W(2_n) - 4_n L(1_n)$, when $K = 2$.

Hirai's suggestion: It is impossible to have $\varphi_0^{S_n^*} \geq W(1_n)$ in any general Cournot markets, because

$$\begin{aligned} & \varphi_0^{S_n^*} \geq W(1_n) \\ \Longleftrightarrow\ & (W(1_n) + 2_n W(2_n) - 2_n L(1_n))/3 \geq W(1_n) \\ \Longleftrightarrow\ & 2_n W(2_n) - 2_n L(1_n) \geq 2_n W(1_n), \end{aligned}$$

which contradicts $W(1_n) \geq W(2_n)$ by $L(1_n) > 0$, when $K = 2$.

4. Assumption (a): General Cournot Markest

As a general property, the Cournot equilibrium price $p = p(t_n)$ satisfies

$$n(p - c) = \frac{p}{\eta(p)} - t_n\varepsilon \text{ if } t_n \leq K, \tag{3}$$

where t_n is the number of licensees. When $t_n \leq K$, by $\eta(p) = -pQ'/Q$ and $Q' = 1/P'$, (3) is rewritten as

$$np + P'Q(p) = nc - t_n\varepsilon. \tag{4}$$

Thus, by (3), (4), and $K = c/\varepsilon\eta(c) = 2$,

$$t_nW(t_n) = -\frac{t_n(p - c + \varepsilon)^2}{P'} = \frac{t_nQ(p)(p - c + \varepsilon)^2}{n(p - c) + t_n\varepsilon}$$
$$= \frac{t_n\eta(p)Q(p)}{p} \cdot (p - 2\varepsilon\eta(c) + \varepsilon)^2$$

where $p = p(t_n)$ is the Cournot equilibrium price.

Consider the total Cournot equilibrium profit of t_n non-licensees. Then, there are $n - t_n$ licensees, and thus (3) is rewritten as

$$n(p-c) = \frac{p}{\eta(p)} - (n - t_n)\varepsilon, \tag{5}$$

where $p = p(n - t_n)$ and $n - t_n$ is the number of licensees. By $\eta(p) = -pQ'/Q$ and $Q' = 1/P'$, (5) is rewritten as

$$np + P'Q(p) = nc - (n - t_n)\varepsilon, \tag{6}$$

If $n - t_n \leq K$, then by (5), (6), and $K = c/\varepsilon\eta(c) = 2$,

$$\begin{aligned} t_n L(n - t_n) &= -\frac{t_n(p-c)^2}{P'} = \frac{t_n Q(p)(p-c)^2}{n(p-c) + (n-t_n)\varepsilon} \\ &= \frac{t_n \eta(p) Q(p)}{p} \cdot (p - 2\varepsilon\eta(c))^2 \end{aligned}$$

where $p = p(n - t_n)$ is the Cournot equilibrium price.

Accoedingly,

$$2_n W(2_n) = \frac{2_n \eta(p(2_n)) Q(p(2_n))}{p(2_n)} \cdot (p(2_n) - 2\varepsilon\eta(c)) + \varepsilon)^2 \,,$$

$$W(1_n) = \frac{\eta(p(1_n)) Q(p(1_n))}{p(1_n)} \cdot (p(1_n) - 2\varepsilon\eta(c) + \varepsilon)^2 \,,$$

$$2_n L(1_n) = \frac{2_n \eta(p(2_n)) Q(p(2_n))}{p(2_n)} \cdot (p(2_n) - 2\varepsilon\eta(c)))^2 \,,$$

where $p(1_n) \geq p(2_n)$, $Q(p(1_n)) \leq Q(p(2_n)$, and $\eta(p(1_n)) \geq \eta(p(2_n))$.

Show that there exists a case where Assumption (a), $2_n W(2_n) - 4_n L(1_n) \geq W(1_n)$, is satisfied.

5. Final Remarks
Farsighted Stability Argument

5. FSS and Open Questions

Farsighted Stability: Harsanyi (1974, Manag Sci), Chwe (1994, JET)
indirect domination is allowed ⇒ negotiation process is analyzed.

- Hirai-Watanabe-Muto (2019): The patent holder's revenue supported by farsighted stable sets with equal treatment of equals widely ranges;

$$0 < x_0 < \max_{t=1,\ldots,n} t(W(t) - L(0)).$$

- an open question: What occurs if the # of firms is very large?
  - Do the farsighted stable sets under some conditions shrink?
  - Is the Aumann-Dréze-Shapley value contained in those farsighted stable sets in a general Cournot market (KOT1992)?
- another open question: What occurs if the patent holder is an incumbent?
  - We should apply absolute maximality (Ray and Vohra, 2019, Econometrica) or history-dependent strongly rational expectation (Dutta and Vohra, 2017, TE) for refining the FSS.

Thanks.