vNM Stable Sets and Aumann-Drèze-Shapley Value in Patent Licensing of Cost-Reducing Technologies: Asymptotic Results in General Cournot Markets

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what we have done

- Shapley value: a concise pricing scheme of information due to its tractability of variables that constitute the data.
    - fair distribution of the total surplus generated by the data
    - It is defined as the average amount of his or her marginal contributions for coalitions that form
- von Neumann-Morgenstern (vNM) stable sets
    - stable standard of behavior shared among players
    - Any outcomes against the standard are rejected and replaced with the outcome which is in accordance with the standard.

- licensing a patented technology of an external patent holder to firms: an information trading game
- Aumann-Drèze-Shapley (ADS) value: the Shapley value extended for games with coalition structures

♣ **Result 1**: When the number of firms that operate in the Cournot markets is relatively small, the ADS value may provide at least as much payoff for the patent holder as a result of a stable standard behavior.

♣ **Result 2**: The same result never holds when the number of firms becomes sufficiently large; vNM stable sets distribute twice as much payoff to the patent holder as the ADS value.

- When the number of firms is limited, various types of the vNM stable sets exist, where the patent holder obtains the other payoffs. We provide an example of such a vNM stable set. (Result 1)
- Those are, however, eliminated as the number of firms in the market tends to infinity. (Result 2)

2. Model: stage (i)

- $N_n = \{1, \dots, n\}$: the set of symmetric firms ($2 \leq n < \infty$)
- player 0: external patent holder ($\{0\} \cup N_n$: the set of players)
- 3-stage game

stage (i): The patent holder selects a set $S_n (\subset N_n)$ of firms for license negotiations.

- Coalition $\{0\} \cup S_n$ forms only for license negotiation.
- $P^{S_n} = \{\{0\} \cup S_n, \{\{i\}\}_{i \in N_n \setminus S_n}\}$: permissible coalition structure

2. Model: stage (ii)

stage (ii): Firms in S_n negotiate license fees with the patent holder and make payments (by means of fixed fee).

- Check the acceptance of payments by each firm after finding the bargaining outcome.
- Analyze the negotiation for each coalition structure P^{S_n}, assuming that all firms in S_n are given a license for simplicity.

2. Model: stage (iii)

stage (iii): Knowing that which firms are licensed, each firm in N_n competes in the market. (Any cartels are prohibited.)

- When t_n firms are licensed, each licensee obtains the gross profit $W(t_n)$ and each non-licensee who uses an old technology obtains the gross profit $L(t_n)$.
- Assume that $W(t_n) > L(0) > L(t_n)\ \forall t_n = 1, \ldots, n-1, (n)$. Negative eternality arises in $L(t_n)$
- Each firm accepts the payment if it is $L(t_n - 1)$ or more.

2. Model: A general Cournot market in stage (iii)

Kamien-Oren-Tauman (1992)

- Each firm i produces q_i unit of a homogeneous commodity with the unit cost of production c. Let $q = \sum_{i \in N_n} q_i$.
- The inverse demand function of the market is $p = P(q)$, where $P(0) > c$. The demand function is denoted by $Q(p)$
    - $P(q)q$ is strictly concave in q.
    - $Q(p)$ is decreasing, differentiable. The price elasticity $\eta(p) = -pQ'/Q$ is non-decreasing in p.
- The patent holder has a patent of a new technology that reduces the unit cost of production from c to $c - \varepsilon$, where $0 < \varepsilon < c$.
- Assume $K = \dfrac{c}{\epsilon\eta(c)} > 1$: non-drastic innovation.

- The Cournot equilibrium gross profits $W(t_n)$ and $L(t_n)$ of each licensee and each non-licensee at stage (iii) are given as

$$W(t_n) = \begin{cases} -\dfrac{(p-c+\varepsilon)^2}{P'} & \text{if } 1 \leq t_n \leq K \\ \dfrac{(p-c+\varepsilon)Q(p)}{t_n} & \text{if } K \leq t_n \leq n, \end{cases}$$

$$L(t_n) = \begin{cases} -\dfrac{(p-c)^2}{P'} & \text{if } 0 \leq t_n \leq K \\ 0 & \text{if } K \leq t_n \leq n. \end{cases}$$

Note that $W(1_n) > \cdots > W(t_n) > \cdots > W(n) > L(0_n) > \cdots L(t_n) \cdots > L(K) = \cdots = L(n-1) = 0$.

Throughout the paper, we assume that licensing the patented technology is beneficial for the industry regardless of the number of licensees, that is, $s_n W(s_n) + (n - s_n)L(s_n) - nL(0_n) > 0$ for any s_n with $1 \leq s_n \leq n$. $\Rightarrow$ We do not consider the social welfare generated by information trading and its maximization.

A sequence of t_n is said to converge to an integer t, if there exists n' such that for all $n > n'$ we have $|T_n| = t$, which is written as

$$t = \lim_{n\to\infty} t_n.$$

Lemma 1

Let $t = \lim_{n\to\infty} t_n$. Then,

(a) If $t \leq K$, then $\lim_{n\to\infty} t_n W(t_n) = t \cdot \varepsilon Q(c)/K$.

(b) If $t > K$, then $\lim_{n\to\infty} t_n W(t_n) = \frac{(c-\varepsilon)Q(p)}{t\eta(p)-1}$.

(c) Either $t \in \mathbb{N}$ or $t = \infty$, $\lim_{n\to\infty} t_n L(n - t_n) = 0$.

2. Model: A bargaining game in stage (ii)

$(\{0\} \cup N_n, v, P^{S_n})$: a game with a coalition structure

$\cdots$ Aumann and Drèze (1974, IJGT)

- $v : 2^{\{0\}\cup N} \to \mathbb{R}$; a characteristic function
  - $v(\{0\}) = v(\emptyset) = 0$.
  - $v(\{0\} \cup T_n) = t_n W(t_n)$ for all nonempty $T_n \subset N_n$.
  - $v(T_n) = t_n L(n - t_n)$ for all nonempty $T_n \subset N_n$.

I^{S_n}: the set of imputations under P^{S_n}, where

$$I^{S_n} = \left\{ \begin{array}{c} x^n = (x_0^n, x_1^n, \ldots, x_n^n) \\ \in \mathbb{R}^{n+1} \end{array} \;\middle|\; \begin{array}{l} x_0^n + \sum_{i\in S} x_i^n = s_n W(s_n), \\ x_0^n \geq v(\{0\}) = 0, \\ x_i^n \geq v(\{i\}) = L(n-1)\ \forall i \in S_n, \\ x_i^n = L(s_n)\ \forall i \in N_n \setminus S_n \end{array} \right\}$$

3. Solution Concepts: vNM stable sets

dominance relation

Let $x, y \in I^{S_n}$. We say that x dominates y via $T_n \subseteq \{0\} \cup N$ if and only if

- $T_n \cap (\{0\} \cup S_n) \neq \emptyset$,
- $\sum_{i \in T_n} x_i \leq v(T_n)$, and
- $x_i > y_i$ for all $i \in T_n \cap (\{0\} \cup S_n)$.

We say that x dominates y if and only if x dominates y via some $T_n \subset \{0\} \cup N_n$.

vNM stable sets

$K^{S_n} \subset I^{S_n}$ is a stable set for a bargaining game $(\{0\} \cup N_n, v, P^{S_n})$ if K^{S_n} satisfies the following conditions.

Internal stability: For any $x^n, y^n \in K^{S_n}$, $x^n \succ y^n$ does not hold.

External stability: For any $x^n \in I^{S_n} \setminus K^{S_n}$, there exists some $x^n \in K^{S_n}$ such that $x^n \succ y^n$.

- For any x_0^n $(0 \leq x_0^n \leq s_n W(s_n))$, define $H^{S_n}(x_0^n) = \{z^n \in I^{S_n} |\ z_0^n = x_0^n\}$.
- Since we are interested in the PH's revenue, we concentrate on a stable set K^S such that $K^{S_n} \subset H^{S_n}(x_0^n)$ for some x_0^n.

3. Solution Concepts: ADS value

Let $\varphi^{S_n}(\in \mathbb{R}^{n+1})$ denote the ADS value of our bargaining game with a coalition structure P^{S_n}, which is represented by

$$\varphi_0^{S_n} = \frac{1}{s_n + 1} \sum_{t=0}^{s_n} t(W(t) - L(n - t)).$$

$\varphi_i^{S_n} = (v(\{0\} \cup S_n) - \varphi_0^{S_n})/s_n$ for all $i \in S_n$, and $\varphi_j^{S_n} = v(\{j\})$ for all $j \in N_n \setminus S_n$.

4. Results: vNM stable sets

Let $H(x_0) = \{y \in I^{S_n} | y_0 = x_0\}$ for each x_0 with $0 \leq x_0 \leq s_n W(s_n)$.

Lemma 2

Let $S_n \neq N$ be nonempty. If

$$s_n W(s_n) + (n - s_n)L(s_n) - nL(0_n) \leq \max_{t_n = 0_n, \ldots, n - s_n} t_n(W(t_n) - L(s_n)), \tag{1}$$

then there exists a vNM stable set V^{S_n} such that $V^{S_n} \subseteq H(s_n W(s_n) + (n - s_n)L(s_n) - nL(0_n))$.[a]

[a] (1) is satisfied if $n \geq 2s_n$

Lemma 3

Let $S_n \subseteq N$ be nonempty. If there exists some $x_0 \in [0, s_n W(s_n)]$ such that some $V^{S_n} \subseteq H(x_0)$ is a vNM stable set, then $x_0 \geq s_n W(s_n) + (n - s_n)L(s_n) - nL(0_n)$.

Let $s_n^* \in \arg\max_{s_n=1_n,\dots,n}(s_n(W(s_n) - L(0_n))$ for each n.

Lemma 4

(a) For each n, let $S_n \subsetneq N_n$ such that $\lim_{n\to\infty} s_n \leq K$. Then, $\lim_{n\to\infty} \varphi_0^{S_n} < \lim_{n\to\infty} s_n W(s_n)$.

(b) $\lim_{n\to\infty} \varphi_0^{S_n^*} = \varepsilon Q(c)/2 = KW(K)/2$.

Proposition 1

Assume that $n \geq 2K$ and $s_n = K$. If $nL(0_n) \geq KW(K)/2$, then $\varphi_0^{S_n} \geq KW(K) - nL(0_n)$.

- $KW(K) - nL(0_n) = s_n W(s_n) - nL(0_n) + (n - s_n)L(s_n)$ when $s_n = K$. The vNM stable set specified by Lemma 3 exists when $n \geq 2s_n$.
- By lemma 1 (c), $\lim_{n\to\infty} nL(0_n) = 0$. $\Rightarrow$ ADS value should be outside of the vNM stable sets/

We describe this statement formally. Let $\nu_0^{S_n}$ denote the infimum of the patent holder's profits in the vNM stable sets when the firms in S_n are licensed. We assume that $\nu_0^{S_n} = -\infty$ if there is no vNM stable set.

Theorem

Let $\bar{S}_n$ be the set of licensees for each n with $|\bar{S}_n| = \bar{s}_n$. Assume that $\lim_{n\to\infty} \bar{s}_n = \bar{s} \in \mathbb{N}$. Then, $\lim_{n\to\infty} \nu_0^{\bar{S}_n} = \lim_{n\to\infty} \bar{s}_n W(\bar{s}_n)$.

There are various types of the vNM stable sets in which the patent holder obtains the other payoffs, but they are all eliminated as the number of firms in the market tends to infinity.

Another type of vNM stable sets. We here omit the subscripts for coalitions.

Proposition 2

Let $n = 2K - 1$. Suppose that $S = \{1, ..., K\}$ are licensed. Assume that $\bar{R} := KW(K) - \max_{m=1,...,K-1} mW(m) - KL(K-1) \geq 0$. Let $r \in [0, \bar{r}]$ where $\bar{r} = \min\left\{\bar{R}/(K-1), KL(K-1)\right\}$. Then,

$$V^r = \left\{ x \in I^S \;\middle|\; \begin{array}{l} x_0 \geq \max_{m=1,...,K-1} mW(m), \\ x_1 \geq KL(K-1), \\ x_i = r \text{ for all } i = 2, ..., K \end{array} \right\}$$

is a vNM stable set.

5. Discussion

Data and variables are easily replicable. ⇒ not scarce ⇒ difficult to put a price on variables and data ⇒ How can we price the data and its constituent variables?

1. "A Model of Pricing Data and Their Constituent Variables Traded in Two-Sided Markets with Resale: A Subject Experiment," by Nanba, Ogawa, Watanabe, Hayashi, Sakaji, *Proc. 2022 IEEE Int'l Conference on Big Data*, IEEE Xplore, 3288-3294, 2022.
2. "Resale-Proof Trades of Data under Budget Constraints: A Subject Experiment," by Ogawa and Watanabe, *Proc. 2023 Int'l Conference on Big Data*, IEEE Xplore, 5665-5673, December 2023.
3. "Resale-Proofness in Sequential Trades of Information," by Naoki Watanabe, mimeo. 2024.

Sequential trades of information is modeled for the the case of incumbent information holders who compete with other agents.

- The prices of variables are exogenously set at the initial round, and they are updated for the next round based on the outcomes of trades made in the current round.
- Resale will stop in the process of sequential trades of information. We can compute the initial price using the backward induction from the last resale.
- The resale-proofness is guaranteed by the “boundary” of the bargaining set. The formula is provided.

Thanks.