An Experimental Study of VCG Mechanism for Multi-unit Auctions: Competing with Machine Bidders

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Oct. 24, 2020 at Hosei Univ. DC on-line meeting

1. Introduction: Motivation

Do we not need to induce bidders to report their true preferences in order to attain the allocative efficiency?

- Takahashi, Izunaga, Watanabe (2019, EIER) examined how VCG mechanism works in multi-unit auctions.
  - The efficiency rates were, again, more than 90% in any sessions. (ref. Takahashi, Izunaga, Watanabe, 2018, ORD)
  - The rates of 95% approximately truth-telling bids were, however, 31.8-43.1% in those sessions.
- Many efficient allocations were not generated even by approximately truth-telling bids.

1. Introduction: Question

When subjects compete with human bidders, it may be difficult for them to realize that truth-telling is their dominant strategy.

Main Hypothesis: When subjects compete with truth-telling machine bidders in multi-unit auctions, the VCG induces subjects to choose truth-telling bids more frequently, and efficient allocations are observed more frequently, as compared to the situation where they compete with human bidders.

1. Introduction: Result

Main Result: Main Hypothesis was confirmed in approximately truth-telling bids and approximately efficient allocations, when unit valuations are shown to subjects and their unit bids are submitted in multi-unit auctions.

An important factor is appearance of information.

1. Introduction: Appearances of Information

Table: Different appearance of information.

Appearance 1	v_i^k shown; b_i^k bid					
# of units		1	2	3	4	5
bidder i	valuation	80×1	60×2	55×3	43×4	77×5
	bid	70×1	55×2	50×3	43×4	72×5
Appearance 2	$v_i^k \cdot k$ shown; $b_i^k \cdot k$ bid					
# of units		1	2	3	4	5
bidder i	valuation	80	120	165	172	385
	bid	70	110	150	172	360

1. Introduction: Background

- Garey, M. R., Johnson, D. S. (2000): In multi-unit auctions, each of the item allocation problems can be formulated as a multiple-choice knapsack problem, which is NP-hard.
  - Takahashi, S., Shigeno, M. (2011, JSIAM Letters) proposed a polynomial-time 2-approximation algorithm, which is called a GBA (greedy-based algorithm), after Kothari et al. (2005).
  - Takahashi, Izunaga, Watanabe (2018, ORD): How GBA works.
  - Takahashi, Izunaga, Watanabe (2019, EIER): How VCG works.
- Rothkopf, M. H., Pekeč, A., and Harstad, R. M. (1998, Manag Sci): In "combinatorial" auctions, each of the item allocation problem can be formulated as a weighted-set packing problem, which is NP-hard.
- Munkres, (1957, J. SIAM): In multi-object auctions, each of the item allocation problems can be formulated as a maximum-weight matching problem on bipartite graph, which is not NP-hard.

2. The Item Allocation in VCG: Instruction

Table: Example.

# of units		1	2	3
Bidder 1	valuation	80×1	60×2	55×3
	bid	70×1	55×2	50×3
Bidder 2	valuation	40×1	70×2	65×3
	bid	40×1	60×2	65×3

The item will be allocated to bidders such that the total amount of bids is maximized as follows. Find an allocation that maximizes the total amount of bids among all possible allocations; In the example, (0, 0): 0, (1, 1): $70\times1+40\times1=110$, (1, 0): $70\times1=70$, (2, 0): $55\times2=110$, (3, 0): $50\times3=150$, (0, 1): $40\times1=40$, (0, 2): $6\times2=120$, (0, 3): $65\times3=195$, (1, 2): $70\times1+60\times2=190$, (2,1): $55\times2+40\times1=150$. Thus, this auction allocates 3 units to bidder 2. The total amount of bids is 195. When there are two or more allocation in each of which the total amount of bids is maximized, one of those allocations is chosen at random.

2. The Payment Determination in VCG: Instruction

The payments of bidders are determined as follows.

payment of bidder i =
(total amount of bids in the auction that excludes bidder i)
− (total amount of bids in the auction)
+ (bidder i's bid for the unit assigned to i).

In the example,

- payment of bidder $1 = (65 \times 3) - 195 + 0 = 0$,
- payment of bidder $2 = (50 \times 3) - 195 + (65 \times 3) = 150$.

3. Experimental Design: Basic Setup 1

- computerized, using cgi script that was coded in Python.
- 2 sessions for Appearance 1, 2 sessions for Appearance 2
- 20 rounds. (10 draws at random, 10 draws in decreasing order; Engelmann and Grimm (2009)
- 5 units of a virtual item are auctioned off to 3 bidders
- unit valuations are independently and uniformly distributed over the integers between 1 and 200.
- Bids are privately made using non-negative integers.
- unit valuations are privately shown only on his or her computer screen.
- When k units of the item are allocated to bidder i, he or she receives the points in the amount $v_i^k \cdot k$ minus his or her payment, which is shown to bidder i through his or her computer screen.
- No information on the other bidders are shown there.

3. Experimental Design: Basic Setup 2

- In the instruction, how the VCG mechanism works is demonstrated using an example.
- The example is carefully made so that it does not imply the dominant strategy of the auction game.
- Subjects are paid according to the total points they obtain in 6 rounds (3 from the first 10 rounds and 3 from the subsequent 10 rounds) randomly selected by a computer at the end of the session
- 1 point = 1 JPY, 1500 JPY for show-up .
- Subjects play 1 round for practice to familiarize themselves with the software, before proceeding to each set of 10 rounds in the session they participate in.

3. Experimental Design: Session Details

Table: Features of the experimental sessions.

session no.	appearance of info.	display of draws	# of subj.	session date	avg. payment per subject
1	2	at random	8	Jan.22	496.54
1	2	descending			1127.89
2	2	descending	8	Jan.22	876.87
2	2	at random			346.65
3	1	at random	8	Jan.23	600.12
3	1	descending			1285.73
4	1	descending	8	Jan.23	1306.58
4	1	at random			527.61

Note: All the above sessions were conducted in January 2018. The display type of draws used in the first 10 rounds is listed first in each session with the same session number. For a companion experiment, at the university of Tsukuba, 2 sessions for Appearance 1 were conducted in February 2015 and 2 sessions for Appearance 2 were conducted in January 2017, in each session of which 24 subjects participated.

3. Experimental Design: Basic Setup 2

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- The example is carefully made so that it does not imply the dominant strategy of the auction game.
- Subjects are paid according to the total points they obtain in 6 rounds (3 from the first 10 rounds and 3 from the subsequent 10 rounds) randomly selected by a computer at the end of the session
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4. Results: Preparation 1

- We analyze the data taken from the last 5 out of 10 rounds in each display type of draws.
- The data were thus merged for each display type of draws to increase our sample size.

Let $\hat{\boldsymbol{x}} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ be an observed allocation. The rate of efficiency is defined by

$$\frac{\sum_{i \in \mathcal{N}} V_i(\hat{x}_i)}{\text{the optimal value of } (AP)_V}.$$

The rate of seller's revenue (profit) is defined by

$$\frac{\text{the total amount of observed payments}}{\text{the total amount of optimal payments}}.$$

4. Results: Allocative Efficiency

Table: The rates of efficiency.

display of draws	at random		decreasing	
appearance of info.	App. 1	App. 2	App. 1	App. 2
mean	0.9892	0.9351	0.9987	0.9765
st.dev.	0.0204	0.0601	0.0022	0.0251
p-value (perm.)	**0.0108**		**0.0016**	

Note: The p-values for the two-sided permutation test are listed in this table. The null hypothesis is rejected at the 5% significance level.

4. Results: Seller's Revenue

Table: The rates seller's revenue.

display of draws	at random		decreasing	
appearance of info.	App. 1	App. 2	App. 1	App. 2
mean	0.9966	1.0014	1.0101	1.0668
st.dev.	0.0125	0.0693	0.0383	0.1487
p-value (perm.)	0.8311		0.2875	

Note: The p-values for the two-sided permutation test are listed in this table. The null hypothesis is rejected at the 5% significance level.

4. Results: Observation 1

Observation 1: (1) Efficient allocations were on average observed significantly more frequently between Appearance 1 and Appearance 2 for each display type of draws. (2) In seller's revenue there was no significant difference on average between Appearance 1 and Appearance 2 for each display type of draws.

4. Results: Preparation 2

We say that a bid for a unit of the item is approximately truth-telling when it satisfies

$$\frac{|\text{unit valuation} - \text{unit bid}|}{\text{unit valuation}} \leq 0.05$$

and that an allocation is approximately efficient when it satisfies

$$\text{the rate of efficiency} \geq 0.95.$$

4. Results: Truth-Telling

Table: Numbers of approximately truth-telling bids and approximately efficient allocations.

	truth-telling		efficiency	
appearance of info.	App. 1	App. 2	App. 1	App. 2
at random	232	148	78	63
p-value (Fisher)	$<$ **0.0001**		**0.0002**	
descending	214	119	74	60
p-value (Fisher)	$<$ **0.0001**		**0.0023**	

Note: The p-values for the one-sided Fisher exact test are listed in this table. The null hypothesis is rejected at the 5% significance level.

4. Results: Observation 2

Observation 2: For each display of draws, (95%) approximately truth-telling bids were chosen by subjects significantly more frequently and (95%) approximately efficient allocations were observed significantly more frequently in Appearance 1 than in Appearance 2.

4. Results: Machine vs Human Bidders in TT

Table: Numbers of approximately truth-telling bids

display of draws	at random		descending	
opponent bidders	machines	subjects	machines	subjects
App. 1	232/400	517/1200	214/400	450/1200
p-value (Fisher)	$< \mathbf{0.0001}$		$< \mathbf{0.0001}$	
App. 2	148/400	493/1200	119/400	381/1200
p-value (Fisher)	0.1576		0.4933	

Note: The p-values for the two-sided Fisher exact test are listed in this table. The null hypothesis is rejected at the 5% significance level. In Appearance 1, the p-values for the one-sided test are also less than 0.001. The sample size is 400 for this experiment and it is 1200 for a companion experiment, respectively.

4. Results: Machine vs Human Bidders in Eff

Table: Numbers of approximately efficient allocations

display of draws	at random		descending	
opponent bidders	machines	subjects	machines	vs. subjects
App. 1	78	60	74	57
p-value (Fisher)	$<$ **0.0001**		**0.0008**	
App. 2	63	60	60	54
p-value (Fisher)	0.7080		0.3826	

Note: The p-values for the two-sided Fisher exact test are listed in this table. The null hypothesis is rejected at the 5% significance level. In Appearance 1, the p-values for the one-sided test are also less than 0.001.

4. Results: Observation 3

Observation 3: When opponents are truth-telling machine bidders, for each display type of draws in Appearance 1, subjects chose approximately truth-telling bids more frequently and approximately efficient allocations realized more frequently, as compared to the case where opponents were human bidders. There were no such differences for each display type of draws in Appearance 2.

Main Result: Main Hypothesis was confirmed when unit valuations are shown to subjects and their unit bids are submitted in multi-unit auctions (Appearance 1).

Main Result suggests a possibility that in Appearance 1 subjects learn their dominant strategy by practicing with machine bidders in experiments for multi-unit auctions, although the item allocation and payment determination under the VCG mechanism is never intuitively understandable to subjects.

5. Final Remarks: Overvid or Underbid?

- In this experiment (vs. machine bidders), subjects overbid for some units while they underbid for other units.
  - Takahashi, Izunaga, and Watanabe (2019, EIER) reports subjects' underbidding in Appearance 1 and their overbidding in Appearance 2. (vs. human bidders)
  - Kagel, Kinross, and Levin (2001, mimeo.) conducted an experiment in which a human bidder with a flat demand for two units competes against machine bidders each demanding a single unit, and they reported overbidding of each human bidder for both units.
  - Chen and Takeuchi (2010, GEB) reports subjects' underbidding in multi-object auctions with package bidding

5. Final Remarks: Other Literature

- 2 units and 2 bidders: Engelmann and Grimm (2009, EJ) compared the performances of a uniform-price sealed-bid auction, a uniform-price clock auction, static and dynamic Vickrey auctions.
- Kagel and Levin (2001, Econometrica) was a seminal paper to study the demand reduction in uniform-price auctions.